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1.Sets

Important Facts and Formulae

1] **Set:** A collection of well defined objects is called a set. The objects which belongs to the set are called elements or members of the set.

2] **Equal sets**: Two sets A and B are said to be equal if every element of A is an element of B and also every elements of B is an element of A.

3] **Cardinal number of a set:** The number of elements in a finite set A is called cardinal number of set A and is denoted by n(A).

4] **Equivalent sets:** Two sets are said to be equivalent if they contain the same number of elements. i.e. if n(A)= n(B)

5] Equal sets are equivalent but equivalent sets are not always equal.

6] **Finite and infinite sets:** A set which contain a finite number of elements is called a finite set and a set which is not finite is called an infinite set.

7] **Empty or null set:** The set which contains no element at all is called an empty set or null set. It is usually denoted by ϕ.

8] **Subset of a set:** If A and B are two set such that every element of A is also an element of the set B, then A is called subset of B and we write $A \subset B$ or by $A \subseteq B$. Two sets A and B are equal if $A \subseteq B$ and $B \subseteq A$.

9] **Number of subsets of a set:** If A is set of 'n' distinct elements then the number of subsets of A is 2^n.

10] **Symmetric difference of two sets:** It is defined as a set consisting of all those members of A which are not in B or those which are in B but not in A and is denoted by $A \Delta B$.

Thus, $A \Delta B = \{x: x \in A \text{ but } x \notin B\} \cup \{x: x \in B \text{ but } x \notin A) = (A-B) \cup (B-A)$

11] **Properties of algebra of sets:**

a) Commutativity: For any two set A and B:

$A \cup B = B \cup A$ and $A \cap B = B \cap A$

b) Associativity: For any three sets A, B and C,

$(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$

c) Distributivity: For any three sets A, B, C

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

d) De Morgan's law: For any two sets A and B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

14] **Ordered pair:** Given two sets A and B, let $a \in A$ and be $b \in B$. Then the ordered pair of two objects a and b is denoted by (a, b), where 'a' is designated as the first member and b as the second.

15] **Equal order pairs:** Two ordered pairs (a, b) and (c, d) are said to be equal if a = c and b = d

Results: 1. when A and B are disjoint sets, then, $n(A \cup B) = n(A) + n(B)$

2. When A and B are not disjoint then, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

<u>Multiple Choice Questions</u>

1. Which of the following collection is set?

a) The collection of prime numbers b) The collection of good teachers in your college

c) Brillant student in class d) Happy people in a city

2. A set is:

a) collection of objects b) well defined collection of objects

c) list of objects d) group of objects

3. How will you write following set in roster form; B= {x|x is a colour in the rainbow}

a) B = {yellow, orange, red, violet, blue, white, grey}

b) B = {black, green, pink, violet, pink, blue, white, grey}

c) B = {Grey red, yellow, violet, pink, blue, indigo}

d) B = {White, black, pink, orange, blue, grey, green}

4. How will you write following set in set builder form: $H= \{5, 5^2, 5^3, 5^4\}$

a) $H = (x|x = 5^n, n \in N, n \leq 4)$ b) $H = (x|x = 5^n, n \notin N, n \geq 4)$

c) $H = (x|x = n^5, n \notin N, n \leq 4)$ d) $H = (x|x = n^2, n \in N, n \geq 4)$

5. State which of the following set is singleton set

a) $B= \{y|y^2 = 36\}$ b) $A = \{x|\sqrt{x} = 16\}$

c) $C = \{x|x^3 = 8\}$ d) D= {x|x is a colour in a rainbow}

6. Which of the following set is empty?

a) A set of all numbers b) B = {x|x is a capital of India}

c) F = {y|y is a point of intersection of two parallel lines}

d) H = {t|t is a triangle having three sides}

7. Find the union of following pair of sets: A= {a, e, i, o, u}, B = {a, b, c, d}

a) AUB = {a, e, i, o, u, a, b, c, d} b) AUB = {a}

c) AUB = {a, b, c, d} d) AUB = {a, b, c, d, e, i, o, u}

8. Find intersection of the following pair of set: $A = \{x|x \in N, 5 < x < 10)$, $B = \{y|y \in W, 5 \leq x < 10\}$

a) $A \cap B = \{5, 6, 7, 8, 9\}$ b) $A \cap B = \{5, 6, 7, 8, 9, 10\}$

c) $A \cap B = \{6, 7, 8, 9\}$ d) $A \cap B = \{5\}$

9. Let $U = \{x|x=2^n, n \in W, n < 8\}$ be the universal set.

$A = \{ y|y = 4^n, n \in N, n < 4\}$. $B = \{z|z = 8^n, n \in N, n \leq 2\}$ Then what will be value of (A - B)'.

a) (A - B)'= {1, 2, 8, 32, 64, 128} b) (A - B)'= {2, 4, 16, 36, 49}

c) (A - B)'= {2, 16, 49, 64} d) (A - B)'= {1, 2, 16, 32, 128}

10. Let S and P be the two sets such that n (S) = 5, n(SUP) = 9, $n(S \cap P) = 2$, find n(P).

a) n (P) = 4 b) n (P) = 2 c) n (P) = 3 d) n (P) = 6

11. In a class of 50 girls, 35 girls like cooking 25 girls like rangoli as well as cooking, All the girls have at least one of the two hobbies. How many girls like only rangoli?

a) 20 girls b) 10 girls c) 15 girls d) 25 girls

12. If A & B are two sets such that n (A) = 70, n (B) = 60, n (AUB) = 110, then $(A \cap B)$ is equal to

a) 240 b) 100 c) 120 d) 20

13. If A = {1, 2, 3}, B = {3, 4} & C = { 1, 3, 5 }, find $(A \times B) \cap (A \times C)$.

a) {(1, 3), (2, 3), (3, 3)} b) {(1, 4), (2, 3), (3, 3)}

c) {(3, 3), (3, 4), (3, 3)} d) {(1, 5), (2, 3), (1, 4)}

14. If A & B are two sets, then $A \cap (A \cap B)'$ equal

a) A b) ϕ c) B d) None of these

15. In a school of 300 students, every student writes 5 essays & every essay is written by 60 students. The number of essays are

a) at least 30 b) at most 20 c) exactly 25 d) none of these

Answer Keys

1. a	2. b	3. a	4. a	5. b	6. c	7. d	8. c	9.a	10. d
11. c	12. d	13. a	14. b	15. c					

HINTS AND SOLUTIONS

1 Let A be the set of girls who like cooking and B = Set of girls who like rangoli.

$n(A) = 35$, $n(AUB) = 50$, $n(A \cap B) = 25$

$\therefore$ n (AUB) = n(A) + n (B) - $n (A \cap B)$ $\therefore$ 50 = 35+ n (B) - 25 $\therefore$ n (B) = 40.

Out of 40 girls, $n (A \cap B) = 25$

$\therefore$ Number of girls who like only rangoli = n(B) - $n (A \cap B)$ = 40 - 25 = 15.

$\therefore$ Answer is (c).

12. $n (A \cap B) = n (A) + n(B) - n (A \cup B) = 70 + 60 - 100 = 20$.

$\therefore$ Answer is (d).

13. A × B = {(1, 3) (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

A × C= {(1, 1) (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)}

(A x B) ∩ (A x C)= {(1, 3) (2, 3), (3, 3)}

∴ Answer is (a).

14. A ∩ (AUB) ' = A ∩ (A' ∩ B') = (A ∩B') ∩ B' = ϕ ∩ B' = ϕ

∴ Answer is (b).

15. Let x, be the number of essays. If each student writes one essay then number of students = 60x. But each student writes 5 essays.

∴ Number of essays = $\frac{60x}{5}$

Now, $\frac{60x}{5}$ = 300 (given) ∴ x = $\frac{1500}{60}$ = 25

∴ Number of essays = 25

∴ Answer is (b).

2. Surds and Indices

Important Facts and Formulae

I. Laws of Indices:

1] $a^m \times a^n = a^{m+n}$ 2] $\frac{a^m}{a^n} = a^{m-n}$ 3] $(a^m)^n = a^{mn}$

4] $(ab)^n = a^n b^n$ 5] $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ 6] $a^0 = 1$

II. Surds: Let a be a rational number and n be a positive integer such that $a^{\frac{1}{n}} = \sqrt[n]{a}$ is irrational. Then, is called a surd of order n.

III. Laws of Surds:

1] $\sqrt[n]{a} = a^{\frac{1}{n}}$ 2] $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ 3] $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

4] $\left(\sqrt[n]{a}\right)^n = a$ 5] $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ 6] $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

IV. If a number equal to the average is added to a group, their average remains the same.

Multiple Choice Question

1. The value of $(256)^{\frac{5}{4}}$ is:
a) 512 b) 984 c) 1024 d) 1032

2. The value of $\left(\sqrt{8}\right)^{\frac{1}{3}}$ is:
a) 2 b) 4 c) $\sqrt{2}$ d) 8

3. The value of $[(10)^{150} \div (10)^{146}]$ is:
a) 1000 b) 10000 c) 100000 d) 10^5

4. $(2.4\times10^3)(8\times10^{-2}) = ?$
a) 3×10^{-5} b) 3×10^4 c) 3×10^5 d) 30

5. $(1000)^7 \div 10^{18} = ?$
a) 10 b) 100 c) 1000 d) 10000

6. $(0.04)^{-1.5} = ?$
a) 25 b) 125 c) 250 d) 625

7. $(17)^{3.5} \times (17)^? = (17)^8$
a) 2.29 b) 2.75 c) 4.25 d) 4.5

8. $49 \times 49 \times 49 \times 49 = 7^?$
a) 4 b) 7 c) 8 d) 16

9. The value of $(8^{-25} - 8^{-26})$ is:
a) 7×8^{-25} b) 7×8^{-26} c) 8×8^{-26} d) None of these

10. If $5^a = 3125$, then the value of $5^{(a-3)}$ is:
a) 25 b) 125 c) 625 d) 1625

11. If $\sqrt{2^n} = 64$, then the value of n is:
a) 2 b) 4 c) 6 d) 12

12. If m and n are whole numbers such that $m^n = 121$, then the value of $(m-1)^{n+1}$ is:
a) 1 b) 10 c) 121 d) 1000

13. $\frac{1}{1+a^{(n-m)}} + \frac{1}{1+a^{(m-n)}} = ?$

a) 0 b) $\frac{1}{2}$ c) 1 d) a^{m+n}

14. If $2^x = \sqrt[3]{32}$, then x is equal to:

a) 5 b) 3 c) $\frac{3}{5}$ d) $\frac{5}{3}$

15. If $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}}$, then x is equal to:

a) $\frac{1}{5}$ b) $-\frac{1}{5}$ c) $\frac{2}{5}$ d) $-\frac{2}{5}$

16. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is:

a) 0 b) 1 c) $\frac{1}{abc}$ d) abc

17. The largest number from among $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$ and is:

a) $\sqrt{2}$ b) $\sqrt[3]{3}$ c) $\sqrt[4]{4}$ d) All are equal

18. If $x = 5+2\sqrt{6}$, then $\frac{(x-1)}{\sqrt{x}}$ is equal to:

a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $\sqrt{3}$ d) $2\sqrt{3}$

19. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to:

a) 0 b) 2 c) 4 d) 6

20. If a, b, c are real numbers, then the value of $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$ is:

a) abc b) $\sqrt{abc}$ c) $\frac{1}{ab}$ d) 1

Answer Keys

1. c	2. c	3. b	4. b	5. c	6. b	7. d	8. c	9.b	10. a
11. d	12. d	13. c	14. d	15. d	16. b	17. b	18. b	19. c	20. d

HINTS AND SOLUTIONS

1. $(256)^{\frac{5}{4}} = (4^4)^{\frac{5}{4}} = 4^{\left(4 \times \frac{5}{4}\right)} = 4^5 = 1024.$

2. $(\sqrt{8})^{\frac{1}{3}} = \left(8^{\frac{1}{2}}\right)^{\frac{1}{3}} = 8^{\left(\frac{1}{2} \times \frac{1}{3}\right)} = 8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\left(3 \times \frac{1}{6}\right)} = 2^{\frac{1}{2}} = \sqrt{2}$.

3. $(10)^{150} + (10)^{146} = \frac{(10)^{150}}{(10)^{146}} = (10)^{150-146} = 10^4 = 10000.$

4. $(2.4\times10^3) \div (8\times10^{-2}) = \frac{2.4\times10^3}{8\times10^{-2}} = (3 \times 10^4).$

5. $(1000)^7 \div 10^{18} = \frac{(1000)^7}{10^{18}} = \frac{(10^3)^7}{10^{18}} = \frac{10^{3\times7}}{10^{18}} = \frac{10^{21}}{10^{18}} = (10)^{(21-18)} = 10^3 = 1000$.

6. $(0.04)^{-1.5} = \left(\frac{4}{100}\right)^{-1.5} = \left(\frac{1}{25}\right)^{\frac{-3}{2}} = (25)^{\frac{3}{2}} = 5^{\left(2 \times \frac{3}{2}\right)} = 5^3 = 125.$

7. Let $(17)^{3.5} \times (17)^x = 17^8$. Then, $(17)^{3.5+x} = (17)^8$.

$\therefore 3.5 + x = 8 \Leftrightarrow x = (8 - 3.5) \Leftrightarrow x = 4.5.$

8. $49 \times 49 \times 49 \times 49 = (7^2 \times 7^2 \times 7^2 \times 7^2) = 7^{(2+2+2+2)} = 7^8$. So, the correct answer is 8.

9. $8^{8^{-25}-8^{-26}} = \left(\frac{1}{8^{25}} - \frac{1}{8^{26}}\right) = \frac{(8-1)}{8^{26}} = 7 \times 8^{-26}$

10. $5^a = 3125 \Leftrightarrow 5^a = 5^5 \Leftrightarrow a = 5.$

$\therefore 5^{(a-3)} = 5^{(5-3)} = 5^2 = 25.$

11. $\sqrt{2^n} = 64 \Leftrightarrow (2^n)^{\frac{1}{2}} = 2 \Leftrightarrow 2^{\frac{n}{2}} = 2^6 \Leftrightarrow \frac{n}{2} = 6 \Leftrightarrow n = 12.$

12. We know that $11^2 = 121$. Putting m = 11 and n = 2, we get:

$(m-1)^{n+1} = (11-1)^{(2+1)} = 10^3 = 1000.$

13. $\frac{1}{1+a^{(n-m)}} + \frac{1}{1+a^{(m-n)}} = \frac{1}{\left(1+\frac{a^n}{a^m}\right)} + \frac{1}{\left(1+\frac{a^m}{a^n}\right)} = \frac{a^m}{(a^m+a^n)} + \frac{a^n}{(a^m+a^n)} = \frac{(a^m+a^n)}{(a^m+a^n)} = 1.$

14. $2^x = \sqrt[3]{32} \Leftrightarrow 2^x = (32)^{\frac{1}{3}} = (2^5)^{\frac{1}{3}} = 2^{\frac{5}{3}} \Leftrightarrow x = \frac{5}{3}$

15. $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}} \quad \Leftrightarrow 2^x \times (2^3)^{\frac{1}{5}} = 2^{\frac{1}{5}} \quad \Leftrightarrow 2^x \times 2^{\frac{3}{5}} = 2^{\frac{1}{5}} \quad \Leftrightarrow 2^{\left(x+\frac{3}{5}\right)} = 2^{\frac{1}{5}}$

$\Leftrightarrow x + \frac{3}{5} = \frac{1}{5} \quad \Leftrightarrow x = \left(\frac{1}{5} - \frac{3}{5}\right) = -\frac{2}{5}$

16. $a^1 = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz} \therefore xyz = 1.$

17. L.C.M. of 2, 3, 4 is 12.

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = 64^{\frac{1}{12}} = \sqrt[12]{64}$

$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = 3^{\frac{4}{12}} = (3^4)^{\frac{1}{12}} = 81^{\frac{1}{12}} = \sqrt[12]{81}$

$\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = 4^{\frac{3}{12}} = (4^3)^{\frac{1}{12}} = 64^{\frac{1}{12}} = \sqrt[12]{64}$

Clearly, $\sqrt[12]{81}$, i.e. $\sqrt[3]{3}$, is the largest.

18. $x = 5+2\sqrt{6} = 3+2+2\sqrt{6} = \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 + 2 \times \sqrt{3} \times \sqrt{2} = \left(\sqrt{3} + \sqrt{2}\right)^2$

Also, $(x-1) = 4+2\sqrt{6} = 2(2+\sqrt{6}) = 2\sqrt{2}\,(\sqrt{2}+\sqrt{3})$.

$\frac{(x-1)}{\sqrt{x}} = \frac{2\sqrt{2}\,(\sqrt{2}+\sqrt{3})}{(\sqrt{3}+\sqrt{2})} = 2\sqrt{2}.$

19. $3^{x-y} = 27 = 3^3 \Leftrightarrow x-y = 3$ ....(i)

$3^{x+y} = 243 = 3^5 \Leftrightarrow x+y = 5$ ....(ii)

On solving (i) and (ii), we get x = 4.

20. $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = (a^{-1})^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \cdot (b^{-1})^{\frac{1}{2}} \cdot c^{\frac{1}{2}} \cdot (c^{-1})^{\frac{1}{2}} \cdot a^{\frac{1}{2}}$

$= (a^{-1}a)^{\frac{1}{2}} \cdot (b^{-1}b)^{\frac{1}{2}} \cdot (c^{-1}c)^{\frac{1}{2}} = (1)^{\frac{1}{2}} \cdot (1)^{\frac{1}{2}} \cdot (1)^{\frac{1}{2}} = (1\times1\times1) = 1.$

3. Ratio and Proportion

Important Facts and Formulae

I. RATIO: Relation obtained by comparing two quantities by division is known as Ratio.

1] The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as a : b.

2] In the ratio a : b, we call a as the first term or antecedent and b, the second term or consequent.

Ex. The ratio 5: 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

3] Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also 4 : 6 = 2 : 3.

4] A ratio is a number, it has no unit of measurement.

5] Second term 'b' in a : b is always non-zero real number.

6] The ratio of a to b is written as a : b.

II. Properties of Ratio:

1] The ratio remains unchanged if both the terms of the ratio are multiplied or divided by same non-zero real number. $\frac{a}{b}$ and $\frac{ak}{bk}$ are equivalent; and $\frac{a}{b}$ and $\frac{a\div k}{b\div k}$ are equivalent.

2] If a × d > b x c then, $\frac{a}{b} > \frac{c}{d}$, b > 0, d > 0

3] If a x b < b x c then, $\frac{a}{b} < \frac{c}{d}$, b > 0, d> 0

4] If a x d = b x c then $\frac{a}{b} = \frac{c}{d}$, b > 0, d> 0

III. Properties of Equal Ratios:

1] $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$ this property is called Invertendo.

2] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$ (Alternendo)

3] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

4] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

5] $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo - Dividendo)

IV. Theorem on Equal ratios:

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

V. COMPOUNDED RATIOS:

1] The compounded ratio of the ratios (a : b), (c : d), (e : f) is (ace : bdf).

2] Duplicate ratio of (a: b) > (c: d) ⇔ $\frac{a}{b} > \frac{c}{d}$.

3] Sub-duplicate ratio of (a : b) is ($\sqrt{a}$: $\sqrt{b}$)

4] Triplicate ratio of (a : b) is (a^3 : b^3).

5] Sub-triplicate ratio of (a: b) is $\left(a^{\frac{1}{3}} : b^{\frac{1}{3}}\right)$

VI. PROPORTION: The equality of two ratios is called Proportion.

If a: b = c: d, we write, a : b :: c : d and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, a:b :: c:d ⇔ (bxc) = (a×d)

VII. Continued Proportion:

1] a, b, c, d, e..... are said to in continued proportion if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e}$ ....

2] If a, b, c are in continued proportion, then product of extreme is equal to square of mean = $b^2 = ac$.

Solved Examples

1) Express the ratio 25: 75 in the simplest form.

Sol. HCF of 25 and 75 is 25. $25 : 75 = \frac{25}{25} : \frac{75}{25} = 1 : 3$

2) a : b = 2: 3 and b : c = 9 : 17. Find a : b : c.

Sol. a : b = 2 : 3 or a : b = 2 x 3 : 3 x 3 = 6 : 9 and b : c = 9 : 17

3) The ratio of two numbers is 4 : 5. If the sum of these numbers is 70, find the numbers.

Sol. Let the two numbers be 4x and 5x. Then 4x + 5x = 70 $\Rightarrow 9x = 70 \quad \therefore x = \frac{70}{9}$

$\therefore$ First number is $4 \times \frac{70}{9} = \frac{280}{9}$ $\therefore$ Second number is $5 \times \frac{70}{9} = \frac{350}{9}$.

4) The ratio of two quantities is 5 : 9. If the first quantity is 112, find the other quantity.

Sol. Let the other quantity is x, then $5 : 9 = 112 : x \Rightarrow \frac{5}{9} = \frac{112}{x} \Rightarrow x = \frac{112 \times 9}{5} \quad \therefore x = 201.6$

5) Chunnu has Rs 5, Rs 2 and Rs 1 coins in the ratio of 1:2:3 amounting to Rs 120. Find the number of coins of each type.

Sol. Let the number of Rs 5, Rs 2 and Rs 1 coins be x, 2x and 3x respectively.

Total value of these coins = 5 x x + 2 x 2x + 1 x 3x = 12x Then, 12x = 120 $\therefore$ x = 10

Number of Rs 5 coins = 10, Number of Rs 2 coins = 20, Number of Rs 1 coins = 30

6) If 4 is subtracted from each of the ratio 5 : 6, the ratio becomes 4 : 5. Find the numbers.

Sol. Let the required numbers be 5x and 6x, then, $\frac{5x-4}{6x-4} = \frac{4}{5}$, 25x - 20 = 24x – 16, x = 4

The required numbers are 5 x 4 = 20 and 6 × 4 = 24

7) Find the mean proportional to 2(a + b) and 8(a + b).

Sol. Required mean proportional = $\sqrt{2(a+b)8(a+b)}$ = 4(a + b)

8) A mixture contains water and alcohol in the ratio of 3 : 1. If it contains 0.25 L of alcohol, find the quantity of water in the mixture.

Sol. $\frac{3}{1} = \frac{x}{0.25}$, x = 0.75 L

9) The ratio of angles of a quadrilateral is 2:3:4: 6. Find the greatest angle.

Sol. Let the angles of a quadrilateral be 2x, 3x, 4x and 6x.

Then, 2x + 3x + 4x + 6x = 360° ($\because$ Sum of angles of a quadrilateral is 360°)

15x = 360° $\therefore$ x = 24°

Largest angle is 6 × 24° = 144°

10) If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ then find the value of $\frac{x+y+z}{x}$

Sol. Given that $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$. Let $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = k$ $\therefore$ x = k, y = 2k, z = 3k

$\therefore \frac{x+y+z}{x} = \frac{k+2k+3k}{k} = \frac{6k}{k} = 6.$

11) If 2x + 3y : 3x + 4y = 5 : 7. Find x : y.

Sol. $\frac{2x+3y}{3x+4y} = \frac{4}{5} \Rightarrow 14x + 21y = 15x + 20y \Rightarrow x = y \quad \therefore x : y = 1 : 1$

Multiple Choice Questions

1. If x : y = 3 : 4 and y : z = 8 : 9, then find the value of x : y : z

a) 3 : 4 : 5 b) 1 : 2 : 3 c) 7 : 12 : 17 d) 6 : 8 : 9

2. If 4x = 5y and 7y = 9z, then x : y : z =

a) 45 : 36 : 28 b) 44 : 33 : 28 c) 28 : 36 : 45 d) 36 : 28 : 45

3. If 30% of x = 20% of y, then find the value of x : y.

a) 1 : 3 b) 3 : 2 c) 3 : 1 d) 2 : 3

4. The mean proportion of 0.32 and 0.02 is

a) 0.06 b) 0.43 c) 0.32 d) 0.08

5. If AB = 36, which of the following is correct?

a) A : 9 = 4 : B b) 9 : A = 4 : B c) A : 17 = B : 7 d) A : 6 = B : 6

6. A bag contains Rs 1, Rs 0.50 and Rs 0.25 coins in the ratio of 8 : 9 : 11. If the total money in the bag is Rs 366, find the number of Rs 0.25 coins.

a) 264 b) 364 c) 241 d) 245

7. Tanvi obtained 12 marks more than that of Ayushi. If the ratio of their marks is 3 : 4, find the sum of their marks.

a) 96 b) 72 c) 84 d) 108

8. In a school, the ratio of boys and girls is 4 : 5. When 100 girls leave the school, the ratio becomes 6 : 7. How many boys are there in the school?

a) 1800 b) 1200 c) 1000 d) 1500

9. A cat takes 5 leaps of every 4 leaps of dog, but 3 leaps of the dog are equal to 4 leaps of the cat. What is the ratio of the speeds of the cat to that of the dog?

a) 11 : 15 b) 15 : 11 c) 16 : 15 d) 15 : 16

10. A sum of Rs 2186 is distributed among A, B and C. If money given to them is decreased by Rs 26, Rs 28 and Rs 32, respectively, then they have money in the ratio 9 : 13 : 8. What is the amount given to A?

a) Rs 575 b) Rs 640 c) Rs 656 d) Rs 672

11. If $\frac{2x^2+3y^2}{2x^2-3y^2} = \frac{125}{71}$, $\frac{4x+3y}{4x-3y} = ?$

a) $\frac{17}{35}$ b) $\frac{19}{37}$ c) $\frac{23}{19}$ d) $\frac{5}{3}$

12. If 5x - 13y = 3x - 8y, then $\frac{x^2+y^2}{x^2-y^2} = ?$

a) $\frac{29}{21}$ b) $\frac{98}{27}$ c) $\frac{3}{5}$ d) $\frac{19}{37}$

13. If 2, b & 8 are in continued proportion find b.

a) 4 b) 6 c) 8 d) 2

14. Find the fourth proportional to 15, 20, 24.

a) 32 b) 16 c) 18 d) 8

15. $4a^2b$, $8ab^2$, m & $2a^3$ are in proportion, Express 'm' in terms of a and b.

a) $\frac{a^3}{b}$ b) $\frac{a^2}{b}$ c) $\frac{a^4}{b}$ d) $\frac{a^5}{b}$

16. Which number is to be added to each of 4, 10, 12, 24 so that resulting numbers are in proportion?

a) 8 b) 6 c) 4 d) 2

17. What is the fourth proportional of the 14, 21, 4.

a) 6 b) 8 c) 4 d) 2

18. Find x if 4.8, 6.0, x & 8.5 are in proportion.

a) 4.2 b) 3.8 c) 6.8 d) 2.4

19. Five numbers are in continued proportion. The first term is 5 & the last term is 80. Find the numbers.

a) 2, 7, 17, 37, 77 b) 5, 10, 20, 40, 80 c) 15, 20, 30, 60, 120 d) 10, 15, 25, 45, 90

20. Find the value of x in, $\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$

a) 2 or 8 b) 0 or 8 c) 2 or 8 d) 0 or 6

21. If A : B = 5 : 7 and B : C = 6 : 11, then A : B : C is:
a) 55 : 77 : 66 b) 30 : 42 : 77 c) 35 : 49 : 42 d) None

22. If A : B = 3 : 4 and B : C = 8 : 9, then A : C is:
a) 2 : 7 b) 4 : 15 c) 8 : 15 d) 15 : 4

23. If A : B = 8 : 15, B : C = 5 : 8 and C : D = 4 : 5, then A : D is equal to:
a) 1 : 3 b) 3 : 2 c) 2 : 3 d) 1 : 2

24. If A : B : C = 2 : 3 : 4, then $\frac{A}{B} : \frac{B}{C} : \frac{C}{A}$ is equal to:
a) 4:9:16 b) 8:9:12 c) 8:9:16 d) 8:9:24

25. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 6 : 7, then A : B : C : D is:
a) 16:22:30:35 b) 16:24:15:35 c) 16:24:30:35 d) 18:24:30:35

26. If 2A = 3B = 4C, then A : B : C is:
a) 2 : 3 : 4 b) 4 : 3 : 2 c) 6 : 4 : 3 d) 20 : 15 : 2

27. If 2A = 3B and 4B = 5C, then A : C is:
a) 4 : 3 b) 8 : 15 c) 15 : 8 d) 3 : 4

28. The ratio of $4^{3.5} : 2^5$ is same as:
a) 2:1 b) 4:1 c) 7:5 d) 7:10

29. If $\frac{1}{5} : \frac{1}{x} = \frac{1}{x} : \frac{1}{125}$ then the value of x is:
a) 1.5 b) 2 c) 2.5 d) 3.5

30. If 0.75 : x :: 5 : 8, then x is equal to:
a) 1.12 b) 1.20 c) 1.25 d) 1.30

31. If 15% of x = 20% of y, then x : y is:
a) 3 : 4 b) 4 : 3 c) 17 : 16 d) 16 : 17

32. If $x^2 + 4y^2 = 4xy$, then x : y is:
a) 2:1 b) 1:2 c) 1:1 d) 1:4

33. The salaries of A, B, C are in the ratio 2:3:5. If the increments of 15%, 10% and 20% are allowed respectively in their salaries, then what will be the new ratio of their salaries?
a) 3:3:10 b) 10:11:20 c) 23:33:60 d) None

34. If 76 is divided into four parts proportional to 7, 5, 3, 4, then the smallest part is:
a) 12 b) 15 c) 16 d) 19

35. Two numbers are in the ratio 3: 5. If 9 is subtracted from each, the new numbers are in the ratio 12:23. The smaller number is:
a) 27 b) 33 c) 49 d) 55

36. Two numbers are in the ratio 1:2. If 7 is added to both, their ratio changes to 3:5. The greatest number is:
a) 24 b) 26 c) 28 d) 32

37. The ratio of three numbers is 3:4:7 and their product is 18144. The numbers are:
a) 9, 12, 21 b) 15, 20, 25 c) 18, 24, 42 d) None of these

38. Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by Rs 4000, the new ratio becomes 40:57. What is Sumit's present salary?
a) Rs 17,000 b) Rs 20,000 c) Rs 25,500 d) None of these

39. The sum of three numbers is 98. If the ratio of the first to the second is 2: 3 and that of the second to the third is 5 : 8, then the second number is:
a) 20 b) 30 c) 48 d) 58 40

40. Two numbers are respectively 20% and 50% more than a third number. The ratio of the 2 numbers is:

a) 2:5 b) 3:5 c) 4:5 d) 6:7

41. Two whole numbers whose sum is 72 cannot be in the ratio:

a) 5:7 b) 3:5 c) 3:4 d) 4:5

42. Seats of Mathematics, Physics and Biology in a school are in the ratio 5:7:8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

a) 2:3:4 b) 6:7:8 c) 6:8:9 d) None of these

43. In a ratio, which is equal to 3:4, if the antecedent is 12, then the consequent is:

a) 9 b) 16 c) 20 d) 24

44. The prices of a scooter and a T.V. are in the ratio 7:5. If the scooter costs Rs 8000 more than a T.V. set, then the price of a T.V. set is:

a) Rs 20,000 b) Rs 24,000 c) Rs 28,000 d) Rs 32,000

45. The ages of A and B are in the ratio 3: 1. Fifteen years hence, the ratio will be 2: 1. Their present ages are:

a) 30 yrs, 10 yrs b) 45 yrs, 15 yrs c) 21 yrs, 7 yrs d) 60 yrs, 20 yrs

46. The average age of three boys is 25 years and their ages are in the proportion 3: 5: 7. The age of the youngest boy is:

a) 21 years b) 18 years c) 15 years d) 9 years

47. The speeds of three cars are in the ratio 5:4:6. The ratio between the time taken by them to travel the same distance is:

a) 5:4:6 b) 6:4:5 c) 10:12:15 d) 12:15:10

48. Zinc and copper are melted together in the ratio 9:11. What is the weight of melted mixture, if 28.8 kg of zinc has been consumed in it?

a) 58 kg b) 60 kg c) 64 kg d) 70 kg

49. A and B are two alloys of gold and copper prepared by mixing metals in the ratio 7:2 and 7:11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold and copper in C will be:

a) 5:7 b) 5:9 c) 7:5 d) 9:5

50. Which of the following ratios is greatest?

a) 7:15 b) 15:23 c) 17:25 d) 21:29

51. What is the ratio of 2 metres with 150 cm in its simplest form

a) 4:3 b) 3:4 c) 6:7 d) 3:5

52. What is the ratio does the second quantity bears with the first quantity in its simplest form 2.4 kg, 3600 grams.

a) 3:5 b) 4:3 c) 3:2 d) 5:7

53. Present age of Gaurav is 7 years 3 months & present age of Rohit is 12 years 1 month find the ratio of their present ages.

a) 2:3 b) 1:3 c) 3:5 d) 5:7

54. Two numbers are in the ratio 3: 5. If 7 is added to each term then ratio becomes 11: 16. Find numbers.

a) 13, 24 b) 15, 25 c) 16, 26 d) 24, 34

55. What is the ratio of second quantity to first in its simplest form, 52 cm, 117 cm

a) 7:4 b) 9:4 c) 7:5 d) 5:32

56. If the ratio of the two numbers is 3: 5 and their sum is 360 find them.

a) 135 & 225 b) 95 & 185 c) 155 & 245 d) 165 & 255

57. The measures of angles of the quadrilateral ABCD are in the ratio 2:3:4:1 quadrilateral.

a) parallelogram b) rhombus c) trapezium d) Square

58. If $\frac{p}{q} = \frac{5}{4}$ then $\frac{3p+2q}{3p-2q}$ = ?

a) 23:7 b) 43:9 c) 63: 11 d) 23:9

59. If $\frac{a}{b} = \frac{5}{6}$ then $\frac{6a-5b}{2a+3b}$ = ?

a) 5 b) 9 c) 0 d) 6

60. If $\frac{x}{y} = \frac{5}{3}$ then $\frac{2x^2+3y^2}{2x^2-3y^2}$ = ?

a) $\frac{66}{17}$ b) $\frac{77}{23}$ c) $\frac{99}{14}$ d) $\frac{55}{23}$

Answer Keys

1. d	2. a	3. d	4. d	5. a	6. a	7. c	8. b	9. d	10. c
11. b	12. a	13. a	14. a	15. c	16. c	17. a	18. c	19. b	20. b
21. b	22. c	23. b	24. c	25. c	26. c	27. c	28. b	29. c	30. b
31. b	32. a	33. c	34. a	35. b	36. c	37. c	38. d	39. b	40. c
41. c	42. a	43. b	44. c	45. b	46. c	47. d	48. c	49. c	50. d
51. a	52. c	53. c	54. b	55. b	56. a	57. c	58. a	59. c	60. b

HINTS AND SOLUTIONS

1. Given that x:y = 3:4 = (3 × 2):(4 × 2) = 6:8

y:z = 6:9 ∴ x:y:z = 6:8:9

The consequent of the first ratio is equal to the antecedent of the second ratio.

2. Given that 4x = 5y ∴ $\frac{x}{y} = \frac{5}{4}$ Also, 7y = 9z ∴ $\frac{y}{z} = \frac{9}{7}$

∴ x:y = 5:4 = (5 × 9):(4 × 9) = 45: 36

y:z = 9:7 = (9× 4):(7 × 4) = 36:28 ∴ x:y:z = 45:36:28

3. Given that 30% of x = 20% of y ⇒ $\frac{x}{y} = \frac{20}{30} = \frac{2}{3}$ ⇒ x:y = 2:3

4. Let x be the mean proportion of 0.32 and 0.02.

$\frac{0.32}{x} = \frac{x}{0.02}$ ⇒ x^2 = 0.0024 ⇒ x = 0.08

5. AB = 36 ∴ AB = 4 x 9 $\frac{A}{9} = \frac{4}{B}$ A:9 = 4:B

7. Let marks of Tanvi be 3x and marks of Ayushi be 4x. As given in question, 4x - 3x = 12

∴ x = 12 Sum of their marks = 4x + 3x = 7x = 7 x 12 = 84

8. Let the number of boys be 4x and number of girls be 5x. As given in question, $\frac{4x}{5x-100} = \frac{6}{7}$

⇒ 28x = 6 (5x-100) ⇒ 28x = 30x - 600 ⇒ 2x= 600 ∴ x = 300

∴ Number of boys = 4x = 4 × 300 = 1200

9. 4 leaps of cat = 3 leaps of dog⇒ 1 leap of cat = $\frac{3}{4}$ leap of dog

Cat takes 5 leaps for every 4 leaps of dog

∴ Required ratio = (5 × cat's leap):(4 × dog's leap) = (5x $\frac{3}{4}$ dog's leap):(4x dog's leap) = 15:16

12. 5x - 13y = 3x - 8y ∴ 5x - 3x = 13y - 8y ∴ 2x = 5y ∴ $\frac{x}{y} = \frac{5}{2}$ ∴ $\frac{x^2}{y^2} = \frac{25}{4}$

∴ $\frac{x^2+y^2}{x^2-y^2} = \frac{25+4}{25-4}$ .....[Componendo - Dividendo] ∴ $\frac{x^2+y^2}{x^2-y^2} = \frac{29}{21}$ ∴ Answer is (a).

16. Let required number is x.

∴ The numbers (4 + x), (10 + x), (12 + x), (24 + x) are in proportion

$\therefore \frac{4+x}{10+x} = \frac{12+x}{24+x}$ $\therefore$ (4+x)(24+ x) = (10+ x)(12+x) $\therefore$ 96 + 28x + x^2 = 120 + 22x + x^2

$\therefore$ 6x = 24 $\therefore$ x = 4 $\therefore$ Answer is (c).

19. Let five numbers in continued proportion are a, ak, ak^2, ak^3, ak^4, a = 5, ak^4 = 80.

$\therefore \frac{ak^4}{a} = \frac{80}{6} = 16$

$\therefore$ k^4 = 16 $\therefore$ k = 2 $\therefore$ ak = 5 x 2 = 10 $\therefore$ ak^2 = 5 x 4 = 20 $\therefore$ ak^3 = 5 x 8 = 40

$\therefore$ 5, 10, 20, 40, 80 are the numbers. $\therefore$ Answer is (b)

20. $\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$

$\therefore \frac{(x^2+12x-20)-4(3x-5)}{3x-5} = \frac{(x^2+8x+12)-4(2x+3)}{2x+3}$ (Dividendo)

$\therefore \frac{x^2+12x-20-12x-20}{3x-5} = \frac{x^2+8x+12-8x-12}{2x+3}$ $\therefore \frac{x^2}{3x-5} = \frac{x^2}{2x+3}$

For x ≠ 0 satisfy the equation, when x ≠ 0, x^2 ≠ 0

dividing by x^2, $\frac{1}{3x-5} = \frac{1}{2x+3}$ $\therefore$ 3x-5 = 2x + 3 $\therefore$ x=8 $\therefore$ x = 0, or x = 8 $\therefore$ Answer is (b).

21. A:B = 5:7, B:C = 6:11 = $\left(6 \times \frac{7}{6}\right) : \left(11 \times \frac{7}{6}\right) = 7 : \frac{77}{6}$

$\therefore$ A:B:C = 5:7: $\frac{77}{6}$ = 30:42:77.

22. $\left(\frac{A}{B} = \frac{3}{4}, \frac{B}{C} = \frac{8}{9}\right) \Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right) = \frac{3}{4} \times \frac{8}{9} = \frac{2}{3} \Rightarrow$ A:C = 2:3.

23. $\frac{A}{B} = \frac{8}{15}, \frac{B}{C} = \frac{5}{8}$ and $\frac{C}{D} = \frac{4}{5} \Rightarrow \frac{A}{D} = \left(\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}\right) = \left(\frac{8}{15} \times \frac{5}{8} \times \frac{4}{5}\right) = \frac{4}{15} \Rightarrow$ A:D = 4:15.

24. Let A = 2x, B = 3x and C = 4x.

Then, $\frac{A}{B} = \frac{2x}{3x} = \frac{2}{3}, \frac{B}{C} = \frac{3x}{4x} = \frac{3}{4}$ and $\frac{C}{A} = \frac{4x}{2x} = \frac{2}{1} \Rightarrow \frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{2}{1}$ = 8:9:24.

25. A:B = 2:3, B:C = 4:5 = $\left(4 \times \frac{3}{4}\right) : \left(5 \times \frac{3}{4}\right) = 3 : \frac{15}{4}$

and C:D = 6:7 = $\left(6 \times \frac{15}{24}\right) : \left(7 \times \frac{15}{24}\right) = \frac{15}{4} : \frac{35}{8}$

$\Rightarrow$ A:B:C:D = 2:3: $\frac{15}{4} : \frac{35}{8}$ = 16:24:30:35

26. Let 2A = 3B and 4B = 5C $\Rightarrow \frac{A}{B} = \frac{3}{2}, \frac{B}{C} = \frac{5}{4}$

$\Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{3}{2} \times \frac{5}{4}\right) = \frac{15}{8} \Rightarrow$ A:C = 15:8.

28. $\frac{4^{3.5}}{2^5} = \frac{(2^2)^{3.5}}{2^5} = \frac{2^{2 \times 3.5}}{2^5} = \frac{2^7}{2^5} = 2^2 = 4.$ $\therefore$ Required ratio is 4: 1.

29. $\frac{1}{5} : \frac{1}{x} = \frac{1}{x} : \frac{100}{125} \Rightarrow \left(\frac{1}{x} \times \frac{1}{x}\right) = \left(\frac{1}{5} \times \frac{100}{125}\right) = \frac{4}{25} \Rightarrow \frac{1}{x^2} = \frac{4}{25} \Rightarrow x^2 = \frac{25}{4} \Rightarrow x = \frac{5}{2} = 2.5$

30. (x x 5) = (0.75 x 8) $\Rightarrow x = \frac{6}{5}$ = 1.20.

31. 15% of x = 20% of y $\Rightarrow \frac{15x}{100} = \frac{20y}{100} \Rightarrow \frac{x}{y} = \left(\frac{20}{100} \times \frac{100}{15}\right) = \frac{4}{3} \Rightarrow$ x:y = 4: 3.

32. $x^2+4y^2 = 4xy \Leftrightarrow x^2 - 4xy + 4y^2 = 0 \Leftrightarrow (x-2y)^2 = 0 \Leftrightarrow 52x^2 = 117y^2 \Leftrightarrow x=2y \Leftrightarrow \frac{x}{y} = \frac{2}{1}$

33. Let A = 2k, B = 3k and C = 5k.

A's new salary = $\frac{115}{100}$ of 2k = $\left(\frac{115}{100} \times 2k\right) = \frac{23}{10}$ k

B's new salary = $\frac{110}{100}$ of 3k = $\left(\frac{110}{100} \times 3k\right) = \frac{33}{10}$ k

C's new salary = $\frac{115}{100}$ of 5k = $\left(\frac{115}{100} \times 5k\right)$ = 6k

$\therefore$ New ratio = $\frac{23}{10}$ k : $\frac{33}{10}$ k : 6k = 23:33:60.

34. Given ratio = 7:5:3:4, Sum of ratio terms = 19. Smallest part = $\left(76 \times \frac{3}{19}\right)$ = 12

35. Let the numbers be 3x and 5x. Then, $\frac{3x-9}{5x-9} = \frac{12}{23}$ ⇔ 23(3x-9) = 12(5x-9) ⇔ 9x = 99 ⇔ x = 11

∴ The smaller number = (3 x 11) = 33.

36. Let the numbers be x and 2x. Then, $\frac{x+7}{2x+7} = \frac{3}{5}$ ⇔ 5(x+7) = 3(2x+7) ⇔ (x = 14).

∴ Greatest number = 28.

37. Let the numbers be 3x, 4x and 7x. Then, 3x x 4x x 7x = 18144 ⇔ $x^3 = 216$ ⇔ $x^3 = 6^3$

⇔ x = 6. ∴ The numbers are 18, 24 and 42.

38. Let the original salaries of Ravi and Sumit be Rs 2x and Rs 3x respectively. Then,

$\frac{2x+4000}{3x+4000} = \frac{40}{57}$ ⇔ 57(2x+4000) = 40(3x+4000) ⇔ 6x = 68000 ⇔ 3x = 34000

Sumit's present salary = (3x + 4000) = Rs (34000+4000) = Rs 38,000.

39. Let the three parts be A, B, C. Then,

A:B = 2:3 and B:C = 5:8 = $\left(5 \times \frac{3}{5}\right) : \left(8 \times \frac{3}{5}\right)$ = ⇒ A:B:C = 2:3: $\frac{24}{5}$ = 10:15:24 ⇒ B = $\left(98 \times \frac{15}{49}\right)$ = 30

40. Let the third number be x.

Then, first number = 120% of x = $\frac{120x}{100} = \frac{6x}{5}$. second number = 150% of x = $\frac{150x}{100} = \frac{3x}{2}$.

∴ Ratio of first two numbers = $\frac{6x}{5} : \frac{3x}{2}$ = 12x:15x = 4:5.

41. The sum of the ratio terms must divide 72. So, the ratio cannot be 3:4.

42. Originally, let the number of seats for mathematics, Physics and Biology be 5x, x and 8x respectively. Number of increased seats are (140% of 5x), (150% of 7x) and (175% of 8x)

i.e. $\left(\frac{140}{100} \times 5x\right), \left(\frac{150}{100} \times 7x\right), \left(\frac{175}{100} \times 8x\right)$ i.e. 7x, $\frac{21x}{2}$ and 14x.

∴ Required ratio = 7x: $\frac{21x}{2}$:14x = 14x:21x:28x = 2:3:4.

43. We have $\frac{3}{4} = \frac{12}{x}$ ⇒ 3x = 48 ⇒ x = 16 ∴ Consequent = 16.

44. Let the price of a scooter and a T.V. set be 7x and 5x respectively.

Then, 7x - 5x = 8000 ⇔ 2x = 8000 ⇔ x = 4000.

∴ Price of a T.V. set = Rs (7x4000) = Rs 28000.

45. Let the ages of A and B be 3x years and x years respectively.

Then, $\frac{3x+15}{x+15} = \frac{2}{1}$ ⇔ 2x+30 = 3x+15 ⇔ x = 15.

46. Total age of 3 boys = (25×3) years = 75 years. Ratio of their ages = 3:5:7.

Age of the youngest = $\left(75 \times \frac{3}{15}\right)$ years = 15 years.

47. Ratio of time taken = $\frac{1}{5} : \frac{1}{4} : \frac{1}{6}$ = 12:15:10.

48. For 9 kg zinc, mixture melted = (9+11) kg.

For 28.8 kg zinc, mixture melted = ($\frac{20}{9}$ x 28.8) kg = 64 kg.

49. Gold in C = $\left(\frac{7}{9} + \frac{7}{18}\right)$ units = $\frac{7}{6}$ units. Copper in C = $\left(\frac{2}{9} + \frac{11}{18}\right)$ units = $\frac{5}{6}$ units.

∴ Gold : Copper = $\frac{7}{6} : \frac{5}{6}$ = 7:5

50. $\frac{7}{15}$ = 0.466, $\frac{15}{23}$ = 0.652, $\frac{17}{25}$ = 0.68 and $\frac{21}{29}$ = 0.724.

Clearly, 0.724 is greatest and therefore, 21: 29 is greatest.

53. present age of Gaurav = 7 years 3 months = 7 × 12 months + 3 months = 87 months

present age of Rohit = 12 year 1 months = 12 x 12 + 1 month = 145 months

∴ Ratio of their present ages = $\frac{\text{present age of Gaurav}}{\text{present age of Rohit}} = \frac{87}{145} = \frac{29 \times 3}{29 \times 5} = \frac{3}{5}$ = 3: 5. ∴ Answer is (c).

54. $\frac{x}{y} = \frac{5}{3}, \frac{x^2}{y^2} = \frac{25}{9}, \frac{2}{3} \times \frac{x^2}{y^2} = \frac{2}{3} \times \frac{25}{9}$ ⇔ $\frac{2x^2}{3y^2} = \frac{5}{7}$ …… [Multiplying by $\frac{2}{3}$]

$\therefore \frac{2x^2+3y^2}{2x^2-3y^2} = \frac{50+27}{50-27}$ ....[Componendo-Dividendo]

$\therefore \frac{2x^2+3y^2}{2x^2-3y^2} = \frac{77}{23}$

∴ Answer is (b).

4. Linear Equation in Two Variables

Important Facts and Formulae

I Introduction:

The general form of a linear equation in one variable is ax + b = 0, where a and b are real numbers, a ≠ 0 and x is a variable.

A value of the variable which satisfies the given equation is called a solution of the equation. e.g., if we put 3 in the equation 4x = x + 9, then LHS = 4 × 3 = 12 and RHS = 3 +9 = 12.

∴ LHS = RHS.

We know that the solution of the given equation will not change,

i) if the same number is added to or subtracted from both the sides of the equation

ii) if both the sides of the equation are multiplied or divided by the same non-zero number.

II Linear Equation in Two Variables and its Solution

The general form of a linear equation in two variables is ax + by + c = 0, where a, b and c are real numbers with a ≠ 0, b ≠ 0 and x, y are two variables.

e.g., i) x+2y = 5 ii) 2x + 3y = 7

In general equation ax + by + c = 0, if either a = 0 or b = 0, then it becomes a linear equation in one variable.

A solution of linear equation in two variables say x and y, is a pair of values, one for x and the other for y, which satisfies the equation.

A linear equation in two variables has infinite number of solutions.

Consider the equation 3x + 4y = 12.

The solutions of this equation are (4, 0) (0, 3), $(1, \frac{9}{4})$, $(2, \frac{3}{2})$, ….. etc.

III. Solution of Simultaneous Equations by Algebraic Method

Graphical method for solving simultaneous equation has some limitations. e.g.,
i) if the solution of simultaneous equations has large values of the variables, e.g., if (-50, 64) is the solution of certain simultaneous equations, then the graphical method will be inconvenient.
(ii) if the solution of simultaneous equations has fractional value for the variables.
e.g. $\left(\frac{7}{15}, \frac{9}{13}\right)$, then this much accuracy is not possible in graphical method.

We shall study two algebraic methods which will give the correct solution to any simultaneous equations:
1) Elimination by Substitution.

2) Elimination by Equation the Coefficient.

<u>Multiple Choice Questions</u>

1. Find the value of x in 2x + 9 = 5x.
   a) 3 b) 5 c) 6 d) 2
2. What is the solution of the equation $\sqrt{3}$x + 2 = 2$\sqrt{3}$ - 1?
   a) 2 + $\sqrt{3}$ b) 2 - $\sqrt{3}$ c) 1 - $\sqrt{3}$ d) 1 + $\sqrt{3}$
3. What will be the value of y in the following expression: $\frac{y-1}{5} = \frac{y}{3}$
   a) $\frac{3}{2}$ b) $\frac{-2}{3}$ c) $\frac{-3}{2}$ d) $\frac{-1}{3}$
4. What is the solution of equation: x + y = 7 and 2x - y = 20
   a) x = 9, y = -2 b) x = -9, y = 2 c) x = 2, y = 9 d) x = - 2, y = - 9
5. The solution of the following equation 45 = 3t + 5, s - t = $\frac{7}{6}$ is
   a) s = $\frac{3}{2}$ b) s = $\frac{3}{2}$, t = $\frac{1}{3}$ c) s = $\frac{2}{3}$, t = 3 d) s = $\frac{-5}{2}$, t = 2
6. Find the value of x and y in the following expressions: 5x + 8y = 9 and 2x + 3y = 4
   a) 2, 5 b) -2, 5 c) 5, -2 d) -5, -2
7. Solve the following simultaneous equations to find the value of x and y:
   $\sqrt{2}$ x + 3y = $\sqrt{3}$; $\sqrt{3}$x - 3y = $\sqrt{2}$

a) x = 1, y = $\frac{\sqrt{3}-\sqrt{2}}{3}$ b) x = -1, y = $\frac{\sqrt{3}-\sqrt{2}}{3}$

c) x = 1, y = $\frac{\sqrt{3}+\sqrt{2}}{3}$ d) x = 0, y = $\frac{\sqrt{3}}{3}$

8. Find the value of following determinant $\begin{vmatrix} 2 & 6 \\ -1 & 3 \end{vmatrix}$.

a) 10 b) 12 c) 9 d) 8

9. Find the value of following determinant $\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$.

a) $\frac{2}{3}$ b) $\frac{-3}{4}$ c) $\frac{4}{3}$ d) $\frac{-4}{3}$

10. What is the solution of following equations: 3x - 4y = 7 and 5x + 2y = 3

a) x = 1 y = -1 b) x = -1, y = -1 c) x = 1, y = 1 d) x = 0, y = 1

11. The sum of two numbers is 146 and their difference is 18. Find the numbers.

a) 82 & 64 b) 89 & 69 c) 80 & 60 d) 88 & 70

12. Two numbers are in the ratio 3:4. If 8 is added to each numbers, their new ratio becomes 4:5. Find numbers.

a) 30 and 38 b) 32 and 40 c) 24 and 32 d) 20 and 28

13. Three chairs and two tables cost Rs 1850 five chairs and three tables cost Rs 2850. Find the cost of four chairs and one table.

a) 1300 Rs b) 2500 Rs c) 1500 Rs d) 1400 Rs

14. Sunil is three times as old as John. After 3 years sunil will be two and half times as old as john. Find their present ages.

a) Sunil's age = 28 yrs, John's age = 9 yrs b) Sunil's age = 28 yrs, John's age = 13 yrs

c) Sunil's age = 27 yrs, John's age = 9 yrs d) Sunil's age = 30 yrs, John's age = 7 yrs

15. Sum of ages of mother and her daughter is 60 years. After 15 years mother's age will be twice as that of her daughter's age at that time find the present age of daughter.

a) 20 years b) 15 years c) 25 years d) 10 years

Q. Write the solutions sets of the simultaneous linear equations:

16. 2x + 5 = 5y + 6; 3 (2x + y) = 24 - x

a) (3, 1) b) (2, 1) c) (4, 3) d) (2, 6)

17. 7x + 13y = 27; 13x + 7y = 33

a) (3, 1) b) (2, 1) c) (1, 2) d) (1, 3)

18. 57x - 56y = 169; 56x - 57y = 170

a) (1, 2) b) (-1,-2) c)(-1, 2) d) (-2, 1)

19. 23p + 27q = 100; 27p + 23q = 100

a) (2, 3) b) (2, 2) c) (2, 1) d) (2, 0)

20. $\frac{x}{2}$ + 3y = 11; x + y = 20

a) (5, 2) b) (2,5) c) (2, 10) d) (10, 2)

21. $\frac{x}{15} + \frac{y}{12} = 5$; $\frac{x}{12} + \frac{y}{15} = \frac{49}{10}$

a) (5, 6) b) (10, 12) c) (15, 18) d) (30, 36)

22. 3x + 2y = 13; $9x^2 - 4y^2 = 65$

a) (3, 2) b) (2, 3) c) (-3, 2) d) (3, -2)

23. $\sqrt{x} + 2\sqrt{y} = 8$ .....(1) & $3\sqrt{x} - \sqrt{y} = 3$ .....(2)

a) (9, 4) b) (4, 9) c) (9, - 4) d) (- 9, 4)

24. $x^2 + 7 = 4y^2$ .....(1) & x+1 = 2y - 16 .....(2)

a) (-3, -2) b) (2, 3) c) (3, 2) d) (3, -2)

Answer Keys

1. a	2. c	3. c	4. a	5. b	6. c	7. a	8. b	9. d	10. a

11. a	12. c	13. a	14. c	15. b	16. a	17. b	18. c	19. b	20. d
21. d	22. a	23. b	24. c						

HINTS AND SOLUTIONS

9. $\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix} = \frac{7}{3} \times \frac{1}{2} - \frac{5}{3} \times \frac{3}{2} = \frac{7}{6} - \frac{15}{6} = \frac{-8}{6} = \frac{-4}{3}$

13. Let cost of each chair = x Rs and cost of each table = y Rs

∴ 3x +2y = 1850 ......(i) and 5x + 3y = 2850 ......(ii)

Multiplying equation (i) by 3 and equation (ii) by 2. We get,

∴ 9x + 6y = 5550 ......(iii) and 10x + 6y = 5700 ......(iv)

By Subtracting: (iii) - (iv); we get, - x = - 150 i.e. x = 150.

Substitute x = 150 in equation (i),

3(150) + 2y = 1850 ∴ 1450 + 2y= 1850 ∴ y = 700.

Total cost = 4 x 150 + 700 = 1300.

16. 2x + 5 = 5y + 6 ∴ 2x - 5y = 6 – 5 ∴ 2x - 5y = 1 ...... (i)

And, 3 (2x + y) = 24 – x ∴ 6x + 3y = 24 - x ∴ 6x + x + 3y = 24 ∴ 7x + 3y = 24 ...... (ii)

Multiplying equation (i) by 7 and equation (ii) by 2,

we get, 14x - 35y = 7 ......(iii) & 14x + 6y = 48 ...... (iv)

Subtracting equation (iv) from equation (iii), we get,

- 41y = - 41 ∴ y = 1 ∴ answer is (a).

17. 7x + 13y = 27 ..... (i) & 13x + 7y = 33 ....(ii)

Adding equations (i) and (ii), we get, 20x + 20y = 60

By dividing both the sides by 20, we get, x + y = 3 ..... (iii)

Subtracting equation (ii) from equation (i) we get, -6x + 6y = – 6

By dividing both the sides by 6 we get, -x + y = -1 ... (iv)

Adding equation (iii) and (iv), we get, 2y = 2 ∴ y = 1

Substituting y = 1 in equation (iii), we get, x + 1 = 3 ∴ x = 2 ∴ answer is (b).

18. 57x - 56y = - 169 ......(i) & 56x - 57y = -170 ..... (ii)

Adding equation (i) and (ii) we get, 113x - 113y = - 339 ∴ x - y = -3 ......(iii)

Subtracting equation (ii) from equation (i), we get, x + y = 1 ....(iv)

Adding equation (iii) and (iv), we get, 2x = -2 ∴ x = -1

Substituting x = - 1 in equation (iv), we get, - 1 + y =1 ∴ y = 2

∴ solution of the equation is (-1, 2). ∴ answer is (c).

20. $\frac{x}{2}$ + 3y = 11 ∴ x + 6y = 22 .....(i) ... (Multiplying both the sides by 2)

x + 5y = 20 ....(ii)

Subtracting equation (ii) from equation (i), we get, y = 2 .... (iv)

Substituting y = 2 in equation (ii), we get,

x + 5 x 2 = 20 ∴ x + 10 = 20 ∴ x = 10

∴ solution is (10, 2) ∴ answer is (d).

23. $9x^2 - 4y^2 = 65$ ∴ $(3x)^2 - (2y)^2 = 65$ ∴ (3x+2y) (3x - 2y) = 65

Putting the value of (3x + 2y) = 13 in the above equation, we get,

13(3x - 2y) = 65 ∴ 3x - 2y = 5

Adding equation (i) and (iii), we get, 6x = 18 ∴ x = 3

Now solve.... ∴ answer is (a).

24. Multiplying equation (ii) by 2 and adding equation (i) and (ii), we get $7\sqrt{x} = 14$

∴ $\sqrt{x} = 2$ i.e. x = 4.

Now putting the value of x = 4 in equation (i) we get, $\sqrt{4} + 2\sqrt{y} = 8$ ∴ $2\sqrt{y} = 6$

$\sqrt{y} = 3$ i.e. y = 9. ∴ Solution = (4,9). ∴ answer is (b).

25. $x^2 - 4y^2 = -7$ ∴ x - 2y = -1 .... (i) & $(x)^2 - (-2y)^2 = -7$ ∴ (x + 2y) (x − 2y) = −7

Putting the value of (x - 2y) = -1 in the above equation,

-1(x + 2y) = - 7 x + 2y = 7.... (ii) ∴ 2x = 6

Now adding (i) and (ii) we get,

2x = 6 ∴ x = 3

Now solve.... ∴ answer is (c).

5. Quadratic Equations

Important Facts and Formulae

I. **Quadratic equation:** A quadratic equation is defined as the equation $ax^2 + bx + c = 0$ where the left-hand side of the equation is a polynomial of degree 2.

II. **Roots of an equation:** The values of x that satisfy an equation are called roots of the equation.

III. **Roots of quadratic equation:** The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

where, a = coefficient of x^2; b = coefficient of x; c = constant term

IV. **Relation between the roots and coefficients:** For a quadratic equation $ax^2 + bx + c = 0$

(i) sum of roots = $\frac{-b}{a}$ (ii) product of roots = $\frac{c}{a}$

V. **Method of finding quadratic equation:** When the roots are known, the equation is given by
x^2- (sum of roots)x + (products of roots) = 0

<u>Multiple Choice Questions</u>

1. If the sum of the roots of a quadratic equation is 6 and the product of roots is also 6, then equation is
a) $x^2 - 6x + 6 = 0$ b) $x^2 + 6x - 6 = 0$
c) $x^2 - 6x - 6 = 0$ d) $x^2 + 6x + 6 = 0$

2. The value of the expression $16x^2 + 24x + 9$ for $x = \frac{-3}{4}$ is
a) 2 b) 1 c) 0 d) -1

3. The solution of simultaneous equations $2x + y = 8$, $5x - 4y = 7$ is
a) x = 2, y = 4 b) x = 3, y = 2 c) x = 5, y = -2 d) x = 6, y = -4

4. If one root of $x^2 - 4x + k = 0$ is 6 then the value of k is
a) -2 b) 12 c) 2 d) -12

5. The values of x & y that simultaneously satisfy the equations $2x + 3y = 5$ & $7x - 4y = 3$ are
a) -1, 1 b) 1, 0 c) 0, 1 d) 1, 1

6. Which one of the following is not factor of $x^3 - 6x^2 + 11x - 6$
a) (x-3) b) (x-6) c) (x-4) d) (x-1)

7. The system of equations $2x + ky + 5 = 0$ & $x - 3y + 9 = 0$ has unique solution when
a) $K \neq 6$ b) $K \neq -6$ c) K = 6 d) K = -6

8. If the roots of the equation $16x^2 - 2kx + 25 = 0$ are equal, then the value of k is
a) 40 b) - 40 c) ± 20 d) ± 10

9. The roots of equation $2x^2 = 3x + 4$ are
a) $\frac{-3+\sqrt{41}}{4}$ b) $\frac{3+\sqrt{41}}{4}$ c) $\frac{-4+\sqrt{41}}{4}$ d) $\frac{4+\sqrt{41}}{4}$

10. If one root of $kx^2 - 5x + 6 = 0$ is 2, find k.
a) 0 b) 1 c) -1 d) -2

11. What are the roots of equation $2x^2 - 7x + 5 = 0$
a) $\left\{\frac{-5}{2}, 1\right\}$ b) $\left\{\frac{5}{2}, 1\right\}$ c) $\left\{\frac{5}{2}, -1\right\}$ d) $\left\{\frac{-5}{2}, -1\right\}$

12. What are the roots of equation, $x^2 - 6x + 9 = 0$
a) {3, -3} b) {-3, -3} c) {3, 3} d) $\left\{\frac{1}{3}, -3\right\}$

13. What are the roots of equation $x^2 - 2x - 2 = 0$
a) $\{1+\sqrt{3}, 1-\sqrt{3}\}$ [b) $(-1+\sqrt{3}, 1-\sqrt{3})$ c) $(-1+\sqrt{3}, -1-\sqrt{3})$ d) $(1-\sqrt{3}, -1+\sqrt{3})$

Q. The roots of the following quadratic equations are:

14. $x^2 - 6x + 5 = 0$
a) (1, 5) b) (3, 5) c) (5, 7) d) (7, 9)

15. $x^2 + 5x - 14 = 0$
a) (2, 3) b) (3, -5) c) (2, -7) d) (7, -5)

16. $y^2 - 3y - 10 = 0$

a) (-2, -5) b) (-2, 5) c) (-3, 2) d) (3, -2)

17. $3x^2 = 4x$

a) (0, 4) b) (0, 3) c) (4, 3) d) (0, 4/3)

18. $25P^2 - 49 = 0$

a) $\left(\frac{7}{5}, \frac{5}{7}\right)$ b) $\left(\frac{7}{5}, \frac{-5}{7}\right)$ c) $\left(\frac{5}{7}, \frac{-5}{7}\right)$ d) $(5, -7)$

19. $\frac{1}{4}(x+5)^2 = 9$

a) (3, -3) b) (1, -11) c) (4, 3) d) (0, 4/3)

20. $6x^2 + 17x + 12 = 0$

a) $(-8, -9)$ b) $\left(\frac{-3}{2}, \frac{4}{3}\right)$ c) $\left(\frac{-3}{2}, \frac{-4}{3}\right)$ d) $\left(\frac{3}{2}, \frac{4}{3}\right)$

21. $4y^2 - 5y = 0$

a) $\left(\frac{5}{4}, 0\right)$ b) $\left(\frac{-5}{4}, 0\right)$ c) $\left(\frac{4}{5}, 0\right)$ d) $\left(\frac{4}{5}, 1\right)$

22. $\left(m + \frac{1}{2}\right)^2 = 1$

a) $\left(\frac{1}{2}, \frac{3}{2}\right)$ b) $\left(\frac{-1}{2}, \frac{3}{2}\right)$ c) $\left(\frac{1}{2}, \frac{-3}{2}\right)$ d) $\left(\frac{-1}{2}, \frac{-3}{2}\right)$

23. $6n^2 - 5n - 21 = 0$

a) $\left(\frac{7}{3}, \frac{-3}{2}\right)$ b) $\left(\frac{7}{3}, \frac{3}{2}\right)$ c) $\left(\frac{-7}{3}, \frac{3}{2}\right)$ d) $\left(\frac{-7}{3}, \frac{-3}{2}\right)$

24. $2y^2 + 11y + 15 = 0$

a) $(-3, -5)$ b) $(-3, 5)$ c) $\left(-3, \frac{5}{2}\right)$ d) $\left(-3, \frac{-5}{2}\right)$

25. $x^2 + 5x - 2 = 0$

a) (0, 5) b) $\left(\frac{-5+\sqrt{33}}{2}, \frac{-5-\sqrt{33}}{2}\right)$ c) $\left(\frac{-5+\sqrt{27}}{2}, \frac{-5-\sqrt{27}}{2}\right)$ d) $\left(\frac{5}{2}, \frac{-5}{2}\right)$

26. One root of the equation $2x^2 + 3x + k = 0$ is Then value of K is:

a) -2 b) 2 c) -1 d) 1/2

27. The sum of the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is:

a) $\frac{c}{a}$ b) $-\frac{c}{a}$ c) $-\frac{b}{a}$ d) $-\frac{a}{b}$

28. If α, β are the roots of the equation $3x^2 - 7x + 13 = 0$, then α + β is equal to:

a) $-\frac{7}{3}$ b) $\frac{7}{3}$ c) $\frac{3}{7}$ d) 1

29. The product of the roots of the equation $\sqrt{11}x^2 - 5x + 3\sqrt{11} = 0$:

a) 3 b) -3 c) $\frac{5}{\sqrt{11}}$ d) 1

Answer Keys

1. a	2. c	3. b	4. d	5. d	6. c	7. b	8. c	9. b	10. b
11. b	12. c	13. a	14.a	15. c	16. b	17. d	18. b	19. d	20. c
21. a	22. c	23. a	24. d	25. b	26. a	27. c	28. a	29. a	

HINTS AND SOLUTIONS

1. Sum of roots = 6; products of roots = 6 ∴ x^2 - (sum of roots) x + (products of roots) = 0

∴ $x^2 - 6x + 6 = 0$ ∴ Answer is (a)

2. At $x = \frac{-3}{4} = 16x^2 + 24x + 9 = 16\left(\frac{-3}{4}\right)^2 + 24\left(\frac{-3}{4}\right) + 9$

$= 16 \times \frac{9}{16} - 24 \times \frac{3}{4} + 9 = 9 - 18 + 9 = 0$ ∴ The answer is (c)

3. $2x + y = 8$ ......(i) $5x - 4y = 7$ .....(ii)

multiplying (i) by 4 & adding resultant to (ii) we get,

$13x = 39$ ∴ $x = 3$

substituting in (i); $2 \times 3 + y = 8$

∴ $y = 8 - 6 = 2$ ∴ $x = 3, y = 2$

4. Since 6 is a root of $x^2 - 4x + k = 0$

$\therefore 6^2 - 4\times6 + k = 0$ $\therefore 36 - 24 + k = 6$ $\therefore k = -12$

5. $2x + 3y = 5$ .....(i) $7x - 4y = 3$ .....(ii)

multiplying (i) by 4 & (ii) by 3 & adding we get,

$29x = 29$; $x = 1$.

putting this value of x in (i), $2 \times 1 + 3y = 5$ $\therefore 3y = 3$ $\therefore y=1$

$\therefore x = 1, y = 1$.

6. $x = 4$ does not satisfy the given equation.

$\therefore (x-4)$ is not factor of $x^3 - 6x^2 + 11x = 6$

7. $2x + ky + 5 = 0$ .....(i) $x - 3y + 9 = 0$ ......(ii)

by cross multiplication; $\frac{x}{9k+15} = \frac{y}{5-18} = \frac{1}{-6-k}$ $- k - 6 \neq 0$ $\therefore k \neq -6$

8. $16x^2 - 2kx + 25 = 0$ roots are equal, then $b^2 - 4ac = 0$

$\therefore (-2k)^2 - 4 \times 16 \times 25 = 0$

14. $x^2 - 6x + 5 = 0$ $\therefore x^2 - 5x - x + 5 = 0$ $\therefore (x-5)(x-1) = 0$ $\therefore x = 5$ or 1

15. $x^2 + 5x - 14 = 0$ $\therefore x^2 + 7x - 2x - 14 = 0$ $\therefore (x+7)(x-2) = 0$ $\therefore x = -7$ or 2

16. $y^2 - 3y - 10 = 0$ $\therefore y^2 - 5y + 2y - 10 = 0$ $\therefore (y-5)(y+2) = 0$ $\therefore y = 5$ or -2

17. $3x^2 = 4x$ $\therefore 3x^2 - 4x = 0$ $\therefore x(3x - 4) = 0$ $\therefore x = 0$ or $x = \frac{4}{3}$

18. $25p^2 - 49 = 0$ $\therefore (5P+7)(5P-7) = 0$ $\therefore p = \frac{-7}{5}$ or $p = \frac{7}{5}$

19. $(x+5)^2 = 9$

Multiplying both sides by 4, $\therefore (x+5)^2 = 36$ $\therefore (x+5)^2 = \pm6 = 0$

$\therefore (x+5) = \pm6$ $\therefore (x+5) = \pm6 = 0$ $\therefore x + 5 = 6$ or $x + 5 = -6$

$\therefore x = 1$ or $x = -11$

20. $6x^2 + 17x + 12 = 0$ $\therefore 6x^2 + 9x + 8x + 12 = 0$ $\therefore 3x(2x+3) + 4(2x+3) = 0$

$\therefore (2x+3)(3x+4) = 0$ $\therefore x = -3/2$ or $x = -4/3$

21. $4y^2 - 5y = 0$ $\therefore y(4y-5) = 0$ $\therefore y = 0$ or $4y = 5$ $\therefore y = 5/4$

22. $\left(m + \frac{1}{2}\right)^2 = 1$ $\therefore \left(m + \frac{1}{2}\right)^2 = (1)^2$ $\therefore m + \frac{1}{2} = \pm 1$ or $m = -\frac{1}{2} \pm 1$ $\therefore m = \frac{1}{2}$ or $m = \frac{-3}{2}$

23. $6n^2 - 5n - 21 = 0$ $\therefore 6n^2 - 14n + 9n - 21 = 0$

$\therefore 2n(3n-7) + 3(3n-7) = 0$ $\therefore (3n-7)(2n+3) = 0$ $\therefore n = \frac{7}{3}$ or $n = \frac{-3}{2}$

24. $2y^2 + 11y + 15 = 0$ $\therefore 2y^2 + 6y + 5y + 15 = 0$ $\therefore 2y(y+3) + 5(y+3) = 0$

$\therefore (y+3)(2y+5) = 0$ $\therefore y = -3$ or $y = \frac{-5}{2}$

25. $x^2 + 5x = 2$ $\therefore x^2 + 5x + \left(\frac{5}{2}\right)^2 = 2 + \left(\frac{5}{2}\right)^2$ .... Completing the square

$\therefore \left(x + \frac{5}{2}\right)^2 = 2 + \frac{25}{4} = \frac{33}{4}$ $\therefore \left(x + \frac{5}{2}\right)^2 = \left(\sqrt{\frac{33}{2}}\right)^2$ $\therefore x + \frac{5}{2} = \pm\sqrt{\frac{33}{2}}$

$\therefore x = \frac{-5 + \sqrt{33}}{2}$ or $x = \frac{-5 - \sqrt{33}}{2}$

26. $2x^2 + 3x + K = 0$. Putting the value $x = \frac{1}{2}$ in the equation we get,

$2\left(\frac{1}{2}\right)^2 + 3 \times \frac{1}{2} + k = 0$ $\therefore \frac{1}{2} + \frac{3}{2} + k = 0$ $\therefore 2 + k = 0$ $\therefore k = -2$

$\therefore$ answer is (a).

27. $ax^2 + bx + c = 0, a \neq 0$; Sum of roots is $\frac{-b}{a}$ $\therefore$ answer is (c).

28. $3x^2 + 7x + 3 = 0$, then $\alpha + \beta$ is: $\frac{-b}{a} = \frac{-7}{3}$ $\therefore$ answer is (a).

29. $\sqrt{11}x^2 - 5x + 3\sqrt{11} = 0$, then $\alpha.\beta$ is $\frac{c}{a} = \frac{3\sqrt{11}}{\sqrt{11}} = 3$ $\therefore$ answer is (a).

6. Arithmetic Progression

Important Facts and Formulae

I. Sequence:

Consider the following arrangement = 2.

The difference between the 3rd and 2nd numbers = 5 - 3 = 2. Here, we find that the difference between any two consecutive number is 2.

In the second arrangement, the first number is $2=2^1$. The second number is $4=2^2$, the third number is $8 = 2^3$, ...

In the third arrangement the difference between the consecutive numbers is - 3. Thus, a sequence is a collection of numbers arranged in some order and obtained in succession according to some definite rule. The individual numbers forming a sequence are called the terms of the sequence.

II. Arithmetic Progression (A.P.)

Consider the following sequences:

1) 1, 4, 7, 10, 13, ...,

2) 24, 20, 16, 12, 8, ...,

3) 3, 9, 27, 81, 243, ...

13-10 = 10-7 = 7-4 = 4-1 = 3.

Here, the difference between the consecutive numbers is 3, which is constant. Therefore, the sequence is an arithmetic progression and the common difference d = 3.

In the second sequence,

8-12 = 12-16 = 16-20 = 20-24 = -4, which is constant. Therefore, this sequence is an arithmetic progression and the common difference d = - 4.

In the third sequence, the difference between the consecutive numbers is not constant. Therefore, the sequence is not an arithmetic progression.

A sequence in which each term (except the first) differs from the previous one by a constant number, the common difference, is an arithmetic progression.

III. The General Term or nth Term of an A.P.

Let a be the first term and d, the common difference of an A.P.

By the definition of an A.P.

$t_1 = a$, ...(1) $t_2-t_1=d$, ...(2) $t_3-t_2=d$, ...(3) $t_n-t_{n-1}=d$, ...(n)

Adding the above n equalities,

$t_n = a + (n + 1)d$, n=1,2,3,.....

∴ the terms of an A.P. whose first term is a and common difference d are a, a + d, a + 2d, ..., a + (n-1)d,......

Formula to find the nth term of an A.P.: t = a + (n + 1) d, where t is n the nth term, a is the first term and d is the common difference.

IV. The Sum of the First n Terms of an A.P.

The sum of the first n terms of an A.P. is denoted by S. Let a be the first term and d the common difference. Then, the terms of the A.P. are a, a + d, a + 2d, ..., a + (n - 1)d, ...

Here, $t_n = a + (n - 1)d$, $n \epsilon N$. Let us denote $t_n = 1$.

Then, the first n terms of the A. P. are a, a + d, a + 2d, ..., l - 2d, l - d, l.

∴ $S_n = a + (a + d) + (a + 2d), ..., + (l - d) + l$

Also, $S_n = l + (l - d) + (l - 2d), ..., + (a + 2d) + (a + d) + a$.

Adding corresponding terms of the above equations,

$2S_n = (a + l) + (a + l) + (a + l), ..., +(a + 1) + (a+ 1) + (a + 1)$ ..... [n times]

∴ $2S_n = n(a + l)$ ∴ $S_n = (a + l)$

Substituting t_1 for a and t_n for l,

$S_n = (t_1+t_n)$ ... (1)

$t_1 = a$ and $t_n = l = a + (n - 1) d$, ∴ $Sn = [a + a + (n-1)d]$

∴ $Sn = [2a + (n-1) d]$ ... (2)

[**Note:** Use formula (1), when the first and last terms of an A.P. are given]

Multiple Choice Questions

1. Find t_n for the following sequence if they are in arithmetic progression 1, 5, 9, 13
   a) 3n-4 b) 4n-3 c) 6n-5 d) 5n-6
2. Find t_n for following sequence if they are A.P.'s: 24, 21, 18, 15....
   a) 27n-3 b) 23n-7 c) 27-3n d) 23n-7
3. For an A.P., if $t_4 = 20$ and $t_7 = 32$ find a, d and t_n.
   a) a = 6, d = 4, t_n = 4n+4 b) a = 8, d = 4, t_n = 4n+4
   c) a = 4, d = 4, t_n = 4n+4 d) a = 8, d = 6, t_n = 4n+4
4. Which term of the A.P. 8, 11, 14, 17 .... is 758?
   a) 251 b) 242 c) 200 d) 223
5. For an A.P., if $t_1 = 100$, $t_n = 1000$ and n = 10 find S_n.
   a) 5000 b) 5500 c) 6000 d) 6550
6. Find the sum of all natural numbers between 50 and 250. which are divisible by 6.
   a) 4950 b) 9540 c) 5490 d) 9940
7. For an A.P. 't_1' = 20, t_n = 200 and n = 10. Find S_n.
   a) 1000 b) 1430 c) 1010 d) 1100
8. Find t_n in 27, 20, 13, 6
   a) 7-34 n b) 34-7n c) 34n-7 d) 34n-6
9. How many terms are there in the A.P. 201, 208, 215, ..., 369?
   a) 20 terms b) 30 terms c) 25 terms d) 15 terms
10. Find the first term if, d = -7, and $t_{17} = -96$
    a) 16 b) 17 c) 15 d) 14
11. Find the 10th term of the AP. 3, 1, -1, -3....
    a) 15 b) -15 c) 12 d) -12
12. Find the common difference, if a = 100 and $t_{20} = 176$.
    a) 6 b) 2 c) 4 d) 5
13. Find the 7th term of the A.P. 6, 10, 14,.....
    a) 30 b) 12 c) 15 d) 14
14. For an A.P. a = 2, d = 3, find S_{12}
    a) 111 b) 222 c)-222 d)-111
15. If for an A.P. a = 8, d = -2, find S_{30}
    a) 630 b) 360 c) -630 d) - 360
16. 5, 8, 11, 14, 17, ... is an A.P. then common difference and first term is:
    a) 3,5 b) 5,3 c) 0, 1 d) None of these
17. 23rd term of the AP 7, 5, 3, 1, ...is:
    a) – 51 b) 53 c) 37 d) - 37
18. Which term of the AP 5, 8, 11, 14, ... is 320.
    a) 104^{th} b) 105^{th} c) 106^{th} d) 64^{th}
19. The sum of all odd numbers between 100 and 200 is:
    a) 6200 b) 6500 c) 7500 d) 3750
20. The sum of all even natural numbers have between 300 and 400 is:
    a) 16500 b) 17500 c) 169400 d) None of these
21. The arithmetic mean between 14 and 18 is:
    a) 16 b) 15 c) 17 d) 32
22. In an AP 3^{rd} term is 18 and 7^{th} term is 30. The sum of is 17th term is:
    a) 600 b) 510 c) 624 d) None of these

23. The sum of first 7 term of an AP is 10 and that of next 7 term is 27 the common difference:
a) 1/7 b) 7 c) 3 d) 1/3
24. The tenth term of the AP 357, 363, 369 is:
a) 423 b) 417 c) 411 d) None of these
25. How many two digits numbers are there which are divisible by 5:
a) 25 b) 23 c) 27 d) 18
26. The sum of AP is 2, 4, 6, 8, 10, 12:
a) 40 b) 42 c) 44 d) 38
27. The sum of 20 terms of an AP whose first term is 4 and common difference is 3:
a) 650 b) 600 c) 550 d) 700
28. In AP the first term is 2 and last term is 29, then the sum of if x = 10 is:
a) 165 b) 160 c) 155 d) 150
29. If the sum of x of terms of an AP is 525 first term is 3 and last term is 39, then 'x' is:
a) 22 b) 20 c) 25 d) 23
30. If (k+1), 3k and (4k + 2) be any three consecutive terms of an AP, then the value of k is:
a) 3 b) 0 c) 1 d) 2
31. Which term of the AP 8, 11, 14, 17, is .... is 758.
a) 250 b) 251 c) 248 d) 252
32. The 5th and 13th term of an AP are 5 and - 13 respectively the first term of the AP is:
a) 3 b) 14 c) -15 d) 9
33. Which term of the AP 64, 60, 56, 52, ... is zero:
a) 16^{th} b) 17^{th} c) 15^{th} d) 14^{th}
34. How many terms of the AP 3, 6, 9, 12, 15, ... must be taken to make the sum 108:
a) 8 b) 12 c) 17 d) 32
35. For AP $t_8 = 36$, then t_{15} is:
a) 540 b) 542 c) 538 d) 536
36. If the 5th and 12th terms of an AP are 14 and 35 respectively, then the first term and common difference are:
a) 2, 3 b) 3,4 c) 1, 2 d) 4,5
37. The Arithmetic mean between 3 and 13 is:
a) 3 b) 8 c) 10 d) 13
38. Which term of the AP 2, 5, 8, ... is 56?
a) 20 b) 21 c) 19 d) 18
39. The sum of 10 terms of the series 3, 8, 13, 18, ... is:
a) 255 b) 280 c) 520 d) 750
40. The sum of 12 terms of an AP whose first term 2 and common difference is 3:
a) 221 b) 222 c) 220 d) 223

Answer Keys

1. b	2. c	3. b	4. a	5. b	6. a	7. d	8. b	9. c	10. a
11. b	12. c	13. a	14. b	15. c	16. a	17. d	18. c	19. c	20. b
21. a	22. b	23. a	24. c	25. d	26. b	27. a	28. c	29. c	30. a
31. a	32. b	33. b	34. a	35. c	36. a	37. b	38. c	39. a	40. b

HINTS AND SOLUTIONS

2. $t_1 = 24, t_2 = 21, t_3 = 18, t_4 = 15$ ..
$t_2 - t_1 = 21 - 24 = -3; t_3 - t_2 = 18 - 21 = -3; t_4 - t_3 = 15 - 18 = -3$
$\therefore a = 24, d = -3$
$t_n = a + (n - 1)d = 24 + (n - 1)(-3) = 24 - 3n + 3 = 27 - 3n.$

∴ answer is (c).

3. Let a be the first term d be the common difference of A.P.

$t_n = a + (n - 1)d = a + (4 - 1)d = a + 3d$; $t_4 = 20$;

$a + 3d = 20$ ……(1)

$t_7 = a + (7 - 1)d = a + 6d$; $t = 32$

∴ $a + 6d = 32$ …….(2)

Subtracting equation (i) from equation (ii) ∴ $3d = 12$ ∴ $d = 4$.

Substituting the value of 'd' in equation (1) $a + 3 \times 4 = 20$ ∴ $a = 8$

$t_n = a + (n-1)d = 8 + (n-1) \times 4 = 8 + 4n - 4 = 4n+4$.

$a = 8$, $d = 4$, $t_n = 4n+4$

∴ answer is (b).

6. The natural numbers between 50 & 250 divisible by 6 = 54, 60, 66 246

$a = t_1 = 54$; $d=6$

$t_n = 246$ and $t_n = a + (n-1)d$ ∴ $246 = 54 + (n-1) \times 6$ ∴ $246 = 48 + 6n$ ∴ $n = 33$.

$S_n = \frac{n}{2}(t_1+t_n) = \frac{33}{2}(54 + 246) = \frac{33}{2} \times 300 = 33 \times 150 = 4950$. ∴ answer is (a).

9. Let there be 'n' terms in the given A.P.

$t_n = 369$, $a = 201$, $d = 208 - 201 = 7$.

$t_n = a + (n-1)d$ ∴ $369 = 201 + (n-1) \times 7$ ∴ $369 - 201 = 7(n - 1)$

∴ $7(n - 1) = 168$, $n - 1 = 24$, $n = 25$

∴ answer is (c).

11. $a = 3$; $d = 1 - 3 = -1 -1 = -3 - (n-1) = -2$

$t_n = a + (n-1)d$ ∴ $t_{10} = 3 + (10-1)(-2) = -15$

∴ answer is (b).

12. $a = 100$, $t_{20} = 176$

$t_n = a + (n-1)d$ ∴ $t_{20} = a + (20-1)d$ ∴ $176 = 100 + 19d$ ∴ $19d = 176-100$ ∴ $19d = 76$ ∴ $d = 4$

∴ answer is (c).

15. $a = 8$, $d = -2$, $n = 30$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{30}{2}[2 \times 8 + (30 - 1)(-2)] = 15[16 + 29 \times (-2)] = -630$.

∴ answer is (c).

16. $d_2 - d_1 = 8 - 5 = 3$, first term is 5.

17. $d_2 - d_1 = 5 - 7 = -2$, $a = 7$

$t_n = a + (n-1)d = 7 + (23 - 1) \times -2 = 7 - 44 = -37$

∴ answer is (d).

18. $a = 5$, $d_2 - d_1 = 8 - 5 = 3$, $t_n = 320$

$t_n = a + (n-1)d$ ∴ $320 = 5 + (n-1)3$ ∴ $320 = 5 + 3n - 3$

∴ $320 - 2 = 3n$ ∴ $318 = 3n$ ∴ $106 = n$

∴ answer is (c).

19. $a = 101$, $d = 103 - 101 = 2$, $n = 199$

$t_n = a+(n-1)d$ ∴ $199 = 101+(n-1)\times 2$ ∴ $199 = 101 + 2n - 2$

∴ $199 = 2n+99$ ∴ $199 - 99 = 2n$ ∴ $100 = 2n$ ∴ $50 = n$

Now, $S_n = \frac{n}{2}(t_1+t_n)$ $S_{50} = \frac{50}{2}[101+199] = 25 \times 300 = 7500$

∴ answer is (c).

20. $a = 301$, $d = 303 - 301 = 2$

$t_n = a + (n-1)d$ ∴ $399 = 301 + (n-1)2 = 301 + 2n - 2 = 299 + 2n$

∴ $399 - 299 = 2n$ ∴ $n = 50$

Then, $S_{50} = \frac{50}{2}[301 + 399] = 25 \times 700 = 17500$

$\therefore$ answer is (b).

21. $\frac{14+18}{2} = 16$ $\therefore$ answer is (a).

22. Let 'a' be the first term and 'd' be the common difference of the A.P.

$t_n = a+(n-1)d$ $\therefore t_3 = a+(3-1)d$ $\therefore t_3 = a+2d$ $\therefore a+2d = 18$ .....(1)

And, $t_7 = a+(n-1)d$ $\therefore t_7 = a+(7-1)d$ $\therefore t_7 = a+6d$ $\therefore a+6d=30$ .....(2)

$a + 2d = 18$ ..... (1)

$a + 6d = 30$ ...... (2)

\- - -

$-4d = -12$

$d = 3$

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$ $\therefore S_n = \frac{17}{2}[2 \times 6 + (17-1)3]$

$\therefore S_n = \frac{17}{2}[12+48]$ $\therefore Sn = \frac{17}{2} \times \frac{60}{1} = 17 \times 30 = 510$

$\therefore$ answer is (b).

23. Given $S_7 = 10$ and $S_{14} = 27$

$S_7 = \frac{7}{2}[2a + 6d] = 10$ and $S_{14} = \frac{14}{2}[2a + 13d] = 27$

$\therefore S_7 = 7a + 21d = 10$ and $S_{14} = 14a + 91d = 27$ $d = \frac{1}{7}$

$\therefore$ answer is (a).

24. $d = 363 - 357 = 6$; $a = 363$

10^{th} term $= a + (n-1)d = 357 + (10-1)6 = 357 + 54 = 411$

$\therefore$ answer is (c).

25. Two digit number divisible by 5 are 10, 15, 20, 25, ... 95.

$a = 10$, $d = 5$

$t_n = a + (n-1)d$ $\therefore 95 = 10 + (n-1)d$ $\therefore 90 = 5n$ $\therefore n = 18$

$\therefore$ answer is (d).

26. Here $a = 2$ and $d = 4 - 2 = 2$ and $n = 6$

$S_6 = \frac{6}{2}[2a+(n-1)d]$ $\therefore S_6 = \frac{6}{2}[2 \times 2 + 5 \times 2]$ $\therefore S_6 = 3[4 + 10] = 42$

$\therefore$ answer is (b).

27. Here $a = 4$ and $d = 8$ and $n = 20$

$S_{20} = \frac{20}{2}[2 \times 4 + 19 \times 3]$ $\therefore S_{20} = 10[8 + 57] = 650$

$\therefore$ answer is (a).

28. Here $a_1 = 2$, $a_{10} = 29$, $n = 10$

$S_{10} = \frac{n}{2}[a_1 + a_{10}]$ $\therefore S_{10} = \frac{10}{2}[2 + 29]$ $\therefore S_{10} = 5 \times 31 = 155$

$\therefore$ answer is (c).

29. Here $S_n = 525$, $a_1 = 3$, $a_n = 39$

$S_n = \frac{n}{2}[a_1 + a_n]$ $\therefore 525 = \frac{n}{2}[3 + 39]$ $\therefore 525 = 21n$ $\therefore 25 = n$

$\therefore$ answer is (c).

30. $3k - (k + 1) = (4k + 2) - 3k$ $\therefore 2k - 1 = k + 2$ $\therefore k = 3$

answer is (a).

32. Let A.P. be a, $a + d$, $a + 2d$

$\therefore t_5 = a + (5 - 1)d$ and $t_{13} = a + (13 - 1)d$ $\therefore a=14$

$\therefore$ answer is (b).

33. $a = 64$, $d = (60 - 64) = -4$, $t_n = 0$

$\therefore t = a + (n-1)d$

$\therefore 0 = 64 + (n - 1) - 4$ $\therefore 0= 64 - 4n + 4$ $\therefore -68 = -4n$ $\therefore 17 = n$

34. Here $a = 3$, $d = 3$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ $108 = \frac{n}{2}[6 + 3(n - 1)]$

$\therefore = n^2 + n - 72 = 0$ $\therefore = (n + 9)(n - 8) = 0$ $\therefore n = 8$

$\therefore$ answer is (a).

37. A. M. $= \frac{3+13}{2} = \frac{16}{2} = 8$ $\therefore$ answer is (b).

38. Here $a = 2$, $d = 3$ and $t_n = 56$ then $n =$?

$\therefore t_n = a + (n-1)d$ $\therefore 56 = 2 + (n-1)3$ $\therefore 56 = 2 + 3n - 3$ $\therefore 57 = 3n$

$\therefore 19 = n$ $\therefore$ answer is (c).

39. Here $a = 3$, $d = 5$, $n = 10$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ $\therefore S_n = \frac{10}{2}[2x3 + (10-1)\times 5]$

$\therefore Sn = 5[6 + 45] = 5 \times 51 = 255$

40. Here $a = 2$, $d = 3$, $S_{12} = \frac{12}{2}[3 \times 12 + 1] = 6 \times 37 = 222$

$\therefore$ answer is (b).

7. Probability

Important Facts and Formulae

I **Experiment:** An operation which can produce some well-defined outcomes is called an experiment.

II. **Random Experiment:** An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples of Performing a Random Experiment:

1] Rolling an unbiased dice.

2] Tossing a fair coin.

3] Drawing a card from a pack of well-shuffled cards.

4] Picking up a ball of certain colour from a bag containing balls of different colours.

Details:

1] When we throw a coin. Then either a Head (H) or a Tail (T) appears.

2] A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

3] A pack of cards has 52 cards.

It has 13 cards of each suit, namely Spades, Clubs, Hearts and Diamonds.

Cards of spades and clubs are black cards. Cards of hearts and diamonds are red cards.

There are 4 honours of each suit. These are Aces, Kings, Queens and Jacks. These are called face cards.

III. **Sample Space:** When we perform an experiment, then the set S of all possible outcomes is called the Sample Space.

Examples of Sample Spaces:

1] In tossing coin, S = {H, T}.

2] If two coins are tossed, then S = {HH, HT, TH, TT}.

3] In rolling a dice, we have, S = {1,2,3,4,5,6}.

IV. **Event:** Any subset of a sample space is called an event.

V. **Probability of Occurrence of an Event:**

Let S be the sample space and let E be an event.

Then, E ⊆ S. ∴ P (E) = $\frac{n(E)'}{n(s)}$

VI. **Results on Probability:**

1] P(S) = 1 2] 0 ≤ P (E) ≤ 1 3] P (ϕ) = 0

4] For any events A and B, we have: P (AUB) = P(A) + P(B) - P(A⋂B).

5] If A denotes (not-A), then P $(\bar{A})$ = 1 - P(A).

<u>Multiple Choice Questions</u>

1. A coin is tossed twice. The probability of getting exactly one head and at least one head is

a) $\frac{1}{2}, \frac{3}{4}$ b) $\frac{2}{3}, \frac{1}{4}$ c) $\frac{1}{4}, \frac{4}{5}$ d) $\frac{1}{2}, \frac{2}{3}$

2. Three unbiased coins are tossed. The probability of getting at most two heads is

a) $\frac{1}{4}$ b) $\frac{3}{8}$ c) $\frac{7}{8}$ d) $\frac{1}{2}$

3. Two dice are thrown simultaneously. the probability of getting numbers whose sum is 6 or 8

a) $\frac{7}{18}$ b) $\frac{2}{9}$ c) $\frac{5}{36}$ d) $\frac{5}{18}$

4. A bag contains six black balls and eight white balls. One ball is drawn at random. What is the probability that the ball drawn is white?

a) $\frac{4}{7}$ b) $\frac{3}{4}$ c) $\frac{4}{5}$ d) $\frac{1}{8}$

5. A card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a

a) $\frac{4}{13}$ b) $\frac{3}{13}$ c) $\frac{2}{13}$ d) $\frac{1}{13}$

6. If the probability of occurrence of an event is 1/5, then the probability of non-occurrence of the event is

a) $\frac{5}{6}$ b) $\frac{1}{6}$ c) $\frac{4}{5}$ d) $\frac{6}{5}$

7. Ram & Shyam are friends. What is the probability that both will have the same birthday? (Ignoring a leap year)

a) $\frac{1}{365}$ b) $\frac{365}{364}$ c) $\frac{364}{365}$ d) None of these

8. Three unbiased coins are tossed. What is the probability of getting at most 2 heads?

a) $\frac{3}{5}$ b) $\frac{2}{5}$ c) $\frac{5}{8}$ d) $\frac{7}{8}$

9. In a simultaneous throw of two dice, what is the probability of getting a total of 7?

a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{3}{5}$ d) $\frac{5}{6}$

10. In box, there are 8 red, 7 blue, and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

a) $\frac{2}{21}$ b) $\frac{5}{21}$ c) $\frac{6}{21}$ d) $\frac{8}{21}$

11. In a simultaneous throw of two coins, the probability of getting at least one head is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$

12. Three unbiased coins are tossed. What is the probability of getting at least 2 heads?

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{8}$

13. Three unbiased coins are tossed. What is the probability of getting at most two heads?

a) $\frac{3}{4}$ b) $\frac{1}{4}$ c) $\frac{3}{8}$ d) $\frac{7}{8}$

14. In a single throw of a dice, what is the probability of getting a number greater than 4?

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{4}$

15. In a simultaneous throw of two dice, what is the probability of getting a total of 7?

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$

16. What is the probability of getting a sum 9 from two throws of a dice?

a) $\frac{1}{6}$ b) $\frac{1}{8}$ c) $\frac{1}{9}$ d) $\frac{1}{12}$

17. In a simultaneous throw of two dice, what is the probability of getting a doublet?

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{2}{3}$ d) $\frac{3}{7}$

18. In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11?

a) $\frac{1}{4}$ b) $\frac{1}{6}$ c) $\frac{7}{12}$ d) $\frac{5}{36}$

19. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3?

a) $\frac{3}{10}$ b) $\frac{3}{20}$ c) $\frac{2}{5}$ d) $\frac{1}{2}$

20. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

a) $\frac{1}{2}$ b) $\frac{2}{5}$ c) $\frac{8}{15}$ d) $\frac{9}{20}$

21. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?

a) $\frac{1}{10}$ b) $\frac{2}{5}$ c) $\frac{2}{7}$ d) $\frac{5}{7}$

22. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card?

a) $\frac{1}{13}$ b) $\frac{4}{13}$ c) $\frac{1}{4}$ d) $\frac{9}{52}$

23. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is

a) $\frac{1}{13}$ b) $\frac{2}{13}$ c) $\frac{1}{26}$ d) $\frac{1}{52}$

24. The probability that a card drawn from a pack of 52 cards will be a diamond or a king is:

a) $\frac{2}{13}$ b) $\frac{4}{13}$ c) $\frac{1}{13}$ d) $\frac{1}{52}$

25. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white?

a) $\frac{3}{4}$ b) $\frac{4}{7}$ c) $\frac{1}{8}$ d) $\frac{3}{7}$

26. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{7}{19}$ d) $\frac{8}{21}$

27. A box contains 4 red balls, 5 green balls and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green?

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\frac{1}{5}$ d) $\frac{7}{15}$

28. A box contains 20 electric bulbs, out of which 4 are defective. Two bulbs are chosen at random from this box. The probability that at least one of these is defective, is:

a) $\frac{4}{19}$ b) $\frac{7}{19}$ c) $\frac{12}{19}$ d) $\frac{21}{95}$

29. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi?

a) $\frac{2}{5}$ b) $\frac{3}{4}$ c) $\frac{3}{5}$ d) $\frac{3}{10}$

30. Two dice are tossed. The probability that the total score is a prime number is:

a) $\frac{1}{6}$ b) $\frac{5}{12}$ c) $\frac{1}{2}$ d) $\frac{7}{9}$

31. A boy throw a die once then the probability of getting a number greater than 4 is:

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{8}$

32. One card is drawn from a well shuffled deck of 52 cards, then the probability that the card on ace is:

a) $\frac{1}{13}$ b) $\frac{2}{13}$ c) $\frac{1}{26}$ d) $\frac{1}{52}$

33. In a throw of a dies, the probability of getting a prime number is:

a) 6 b) $\frac{1}{2}$ c) 2 d) $\frac{3}{2}$

34. A book containing 80 pages is opened at random of the probability that a doublet page is found is:

a) $\frac{1}{80}$ b) $\frac{7}{80}$ c) $\frac{3}{80}$ d) None of these

35. A coin is tossed than the probability of getting a head is:

a) $\frac{1}{2}$ b) 1 c) 0 d) None of these

36. Three coins are tossed simultaneously then the probability of getting almost one tail is:

a) 1 b) $\frac{1}{2}$ c) 3 d) $\frac{1}{3}$

37. A die is thrown, then the probability of the event of getting an even number is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) 2 d) $\frac{3}{2}$

38. A bag contains 5 red, 8 white and 7 black balls. A ball is drawn at known from the bag, then the probability that the red or white ball drawn is:

a) $\frac{11}{20}$ b) $\frac{9}{20}$ c) $\frac{13}{20}$ d) $\frac{7}{20}$

39. A die is thrown then the probability of getting even number is:

a) 1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{6}$

40. A card is drawn from pack of 52 cards the probability that it is either a spade or a king is:

a) $\frac{1}{20}$ b) $\frac{4}{13}$ c) $\frac{17}{52}$ d) $\frac{15}{52}$

41. Tickets numbered from 1 to 25 are mixed up together and then a ticket is drawn at random. What is the probability that the drawn ticket has a prime number?

a) $\frac{7}{25}$ b) $\frac{1}{25}$ c) $\frac{9}{25}$ d) $\frac{11}{25}$

42. A box contains cards marked with the numbers 1 to 25 one card is drawn from this box, then the probability that the number on the card is a perfect square:

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{4}{5}$ d) $\frac{3}{5}$

43. A die is thrown then the probability that an odd number comes up is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$

44. Two dice are thrown then the probability that the sum of the numbers on the dice is divisible by 9 is:

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{3}$

45. A bag contains 3 white and 5 red balls. If a ball is drawn at random, the probability that the drawn ball is red

a) $\frac{3}{8}$ b) $\frac{5}{8}$ c) $\frac{3}{15}$ d) $\frac{5}{15}$

46. An unbiased die is thrown then the probability that the number on the upper most face of the die is a perfect square is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{2}{3}$

47. A coin is tossed then probability of getting a tail is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) None of these

48. An unbiased die is thrown then the probability that the number on the upper most face is divisible by 2 is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{2}{3}$

49. When two coins are tossed then probability of getting one heads is:

a) $\frac{2}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) None of these

50. Two fair coins are tossed in the same time. The probability of getting atleast one tail is:

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) None of these

Answer Keys

1. a	2. c	3. d	4. a	5. a	6. c	7. a	8. d	9. a	10. d
11. d	12. b	13. d	14. b	15. a	16. c	17. a	18. d	19. a	20. d
21. c	22. b	23. c	24. b	25. b	26. d	27. b	28. b	29. a	30. b
31. c	32. a	33. b	34. b	35. a	36. b	37. a	38. c	39. b	40. b
41. c	42. a	43. a	44. c	45. b	46. b	47. a	48. a	49. d	50. c

HINTS AND SOLUTIONS

1. Let S be the sample space. ∴ S = {HH, HT, TH, TT} ∴ n(S) = 4

Let E_1 = Event of getting exactly one head and E_2 = Event of getting at least on tail

∴ E_1 = {HT, TH} ⇒ $n(E_1) = 2$ and E_2 = {HH, HT, TH} ⇒ $n(E_2) = 3$

∴ $P(E_1) = \frac{n(E1)}{n(S)} = \frac{2}{4} = \frac{1}{2}$, $P(E_2) = \frac{n(E2)}{n(S)} = \frac{3}{4}$.

4. Here, $n(S) = {}^{6+8}C_1 = {}^{14}C_1 = 14$ and $n(E) = {}^{8}C_1 = 8$ ∴ $P(E) = \frac{n(E)}{n(S)} = \frac{8}{14} = \frac{4}{7}$

5. Here, $n(S) = {}^{52}C_1 = 52$ and $n(E) = {}^{13}C_1 + {}^{3}C_1 = 13 + 3 = 16$ ∴ $P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$

6. Here, $P(E) = \frac{1}{5}$ ∴ $P(E') = 1 - P(E) = 1 - \frac{1}{5} = \frac{4}{5}$.

7. Here, $n(S) = 365 \times 365$ ∴ $n(E) = 365$ ∴ $P(E) = \frac{n(E)}{n(S)} = \frac{365}{365 \times 365} = \frac{1}{365}$

9. We know that in a simultaneous throw of two dice, n(S) = (6 × 6) = 36

Let E = Event of getting total of 7 = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)} = 6

∴ $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

10. Total number of balls = (8+7+6) = 21

Let E = Event that the ball drawn is neither red nor green = Event that the ball drawn is red

∴ n(E) = 8 ∴ $P(E) = \frac{8}{21}$

11. Here S = {HH, HT, TH, TT}.

Let E = event of getting at least one head = {HT, TH, HH}.

$\therefore$ P(E) = $\frac{n(E)}{n(S)} = \frac{3}{4}$.

12. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at least two heads = {THH, HTH, HHT, HHH}.

$\therefore$ P(E) = $\frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$.

13. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT)}. $\therefore$ P(E) = $\frac{n(E)}{n(S)} = \frac{7}{8}$

14. When a die is thrown, we have S = {1, 2, 3, 4, 5, 6}.

Let E = event of getting a number greater than 4 = {5, 6}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

15. We know that in a simultaneous throw of two dice, n (S) = 6 x 6 = 36.

Let E = event of getting a total of 7 = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

16. In two throws of a die, n (S) = (6 × 6) = 36.

Let E = event of getting a sum 9 = {(3, 6), (4, 5), (5, 4), (6, 3)}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

17. In a simultaneous throw of two dice, n (S) = (6 × 6) = 36.

Let E = event of getting a doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

18. In a simultaneous throw of two dice, we have n (S) = (6 x 6) = 36.

Let E = event of getting a total of 10 or 11 = {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5)}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{5}{36}$

19. Here, S = {1, 2, 3, 4, ..., 19, 20}.

Let E = event of getting a multiple of 3 = {3, 6, 9, 12, 15, 18}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$

20. Here, S = {1, 2, 3, 4, ...,19, 20}.

Let E = event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{9}{20}$.

21. P (getting a prize) = $\frac{10}{(10+25)} = \frac{10}{35} = \frac{2}{7}$

22. Clearly, there are 52 cards, out of which there are 16 face cards.

$\therefore$ P (getting a face card) = $\frac{16}{52} = \frac{4}{13}$.

23. Here n (S) = 52. Let E = event of getting a queen or club or a king of heart.

Then n (E) = 2 $\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

24. Here, n (S) = 52. There are 13 cards of diamond (including one king) and there are 3 more kings. Let E = event of getting a diamond or a king.

Then n (E) = (13+3)= 16. $\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$.

25. Total number of balls = (6+8) = 14. Number of white balls = 8.

P (drawing a white ball) = $\frac{8}{14} = \frac{4}{7}$.

26. Total number of balls = (8+7+6) = 21.

Let E = event that the ball drawn is neither red nor green = event that the ball drawn is red.

n (E) = 8. ∴ P (E) = $\frac{8}{21}$.

27. Total number of balls = (4+5+6) = 15. ∴ n (S) = 15.

Let E_1 = event of drawing a red ball and E_2 = event of drawing a green ball.

Then, E1 ∩ E2 = ϕ. P (E1 or E2) = P(E1) + P(E2) = $\left(\frac{4}{15} + \frac{5}{15}\right) = \frac{9}{15} = \frac{3}{5}$.

28. P (None is defective) = $\frac{^{16}C_2}{^{20}C_2} = \left(\frac{16 \times 15}{2 \times 1} \times \frac{2 \times 1}{20 \times 19}\right) = \frac{12}{19}$

P (at least one is defective) = $\left(1 - \frac{12}{19}\right) = \frac{7}{19}$

29. P (E) = $\frac{30}{100} = \frac{3}{10}$, P(H) = $\frac{20}{100} = \frac{1}{5}$ and P (E ∩ H) = $\frac{10}{100} = \frac{1}{10}$

P (E or H) = P(E∪H) = P(E) + P(H) - P (E∩H) = $\left(\frac{3}{10} + \frac{1}{5} - \frac{1}{10}\right) = \frac{4}{10} = \frac{2}{5}$.

30. Clearly, n (S) = (6 × 6) = 36.

Let E = Event that the sum is a prime number.

Then, E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3),) (5, 2), (5, 6), (6, 1), (6, 5)}.

∴ n (E) = 15. ∴ P (E) = $\frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

31. Here, let E be the event getting a number greater than 4. The number of possible outcomes is six: 1, 2, 3, 4, 5 and 6, and the outcomes favourable to E are 5 and 6.

Therefore, the number of outcomes favourable to E is 2.

So, P (E) = P(number greater than 4) = $\frac{2}{6} = \frac{1}{3}$

32. Well-shuffling ensures equally likely outcomes,

There are 4 aces in a deck. Let E be the event 'the card is an ace'.

The number of outcomes favourable to E = 4

The number of possible outcomes = 52. Therefore, P (E) = $\frac{4}{52} = \frac{1}{13}$

33. E= {2, 3, 5} ∴ n(E) = 3 & S = {1, 2, 3, 4, 5, 6} ∴ n(S) = 6

∴ P (E) = $\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

34. E= {11, 22, 33, 44, 55, 66, 77} ∴ n (E) = 7

And S = {1, 2, 3, ... 80} ∴ n (S) = 80 ∴ P (E) = $\frac{7}{80}$

35. Let S be the sample space. ∴ S = {H, T} ∴ n(S) = 2.

Let A be the event of getting head. ∴ A = {H} ∴ n (A) = 1 ∴ P (A) = $\frac{1}{2}$

36. Let C be the event of getting atmost one tail. ∴ C= {HHH, HHT, THH, HTH}

∴ n (C) = 4 ∴ P(C) = $\frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$.

37. Let S be the sample space. ∴ S= {1, 2, 3, 4, 5, 6} ∴ n (S) = 6.

Let B be the event of getting an even number ∴ B = {2, 4, 6} ∴ n (B) = 3

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

38. Total balls = 5 red + 8 white + 7 black balls = 20 balls.

P (red or white) = $\frac{\text{No. of favourable outcomes}}{\text{total no.of outcomes}} = \frac{5+8}{20} = \frac{13}{20}$

40. n (S) = 52. Let E = Event of getting a space & F = Event of getting a king of space.

Then, n (E) = 13, n (F) = 4, n (E∩F) = 1

P(E) = $\frac{13}{52} = \frac{1}{4}$ P(F) = $\frac{4}{52} = \frac{1}{13}$. and P(E∩F) = $\frac{1}{52}$

P (E or F) = P(E∩F) ∴ $\left(\frac{1}{4} + \frac{1}{13} - \frac{1}{52}\right) = \frac{16}{52} = \frac{4}{13}$.

41. S = {1, 2, 3, 4, ... 24, 25} & E = {2, 3, 5, 7, 11, 13, 17, 19, 23}

n (S) = 25 and n (E) = 9 P(E) = $\frac{n(E)}{n(S)} = \frac{9}{25}$

42. Let S be the sample space. ∴ S = {1, 2, 3, ... 24, 25) ∴ n(S) = 25

Let B be the event that the number on the card drawn is a perfect square.

∴ B = {1, 4, 9, 16, 25} ∴ n(B) = 5

∴ $P(B) = \frac{n(B)}{n(S)} = \frac{5}{25} = \frac{1}{5}$.

43. Let S be the sample space. Then, S = {1, 2, 3, 4, 5, 6} ∴ n (S) = 6.

Let A be the event where an odd number comes up.

Then, A = {1, 3, 5} ∴ n (A) = 3 ∴ $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

44. The sample space is

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6,), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

This contains 36 sample points. ∴ n (S) = 36.

Let B be the event that the sum of the numbers on the dice is divisible by 9.

Then, B = {(3, 6), (4, 5), (5, 4), (6, 3)} ∴ n(B) = 4 ∴ $P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$.

45. n (E) = 5 & n (S) = 3+5 = 8 ∴ $P(E) = \frac{n(E)}{n(S)} = \frac{5}{8}$

47. A coin is tossed then n (S) = 2 ∴ Let event be a getting tail is n (A) = 1.

∴ $P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$

48. Since n (S) = 6. Let b is event that it is divisible by 2.

∴ B = {2, 4, 6} ∴ n (B) = 3 ∴ $P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

49. If two coins are tossed. ∴ S= {TT, TH, HT, HH} ∴ n (S) = 4

Let E be event that one head ∴ E = {TH, HT} ∴ n (E) = 2

∴ $P(E) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$.

50. Since n (S) = 4 Let A be event that getting atleast one tail.

∴ n(A) = {HT, TH, TT} ∴ n (A) = 3.

∴ $P(A) = \frac{3}{4}$

8. Statistics

Important Facts and Formulae

I. Introduction:

Statistics is the branch of Mathematics which deals with the collection, presentation and analysis of numerical data and draws conclusions on the basis of the same.

Statistical techniques are used in the field of research.

Statistics is used in the study of Economics, Psychology, Physics, Biology, Medicines, etc.

Some basic concepts and the terms:

1] **Data:** Data is a collection of figures or numbers about any subject.

2] **Population:** Population is a set of well defined objects about which statistical information is required.

3] **Sample:** A sample is any finite set of objects drawn from the population, i.e., a sample is a subset of population.

4] **Variate:** A quantitative characteristic is called a variate. A variate is expressed in numbers. There are two types of variates:

A) Discrete variate and

B) Continuous variate

A) Discrete variate takes integral values. For example: Number of children in a family, number of flowers on a tree, etc.

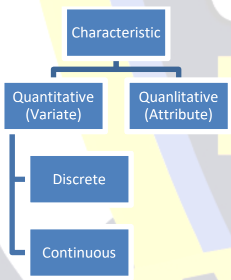
B) Continuous variate takes any real value within a certain interval.

e.g. Rainfall in a city, weight-height of students, etc.

5] **Attribute:** A qualitative characteristic is called an attribute. It cannot be described numerically.

e.g. The population can be divided into categories like, 'male' and 'female', 'Literate' and 'illiterate'.

II. Characteristic:



Basic terms in statistics:

1. **Class:** When the observation are divided into suitable groups. Each of the group is called class.
2. **Class limit:** Each class is bounded by two quantities, called class limits. The quantities on the left side are called lower limit while on the right side are called upper limit.
3. **Frequency:** The total number of observation in each class is frequency.
   Eg. There are 20 observation in class 11-15. Hence frequency is 20.
4. **Class width:** When continuous classes are given, the difference between Upper class limit and lower class limit is known as class width.
   Eg. 5-10, 10-15, 15-20 ,...... class width is 10 - 5 =5.
5. **Class mark:** The middle value of the selected class size or the average of Upper class limit and lower class limit.
   It can be calculated as: Class mark = $\frac{\text{Lower class limit} + \text{Upper class limit}}{2}$

III. Measures of Central Tendency

The number around which the numbers in the data tend to cluster is called measure of central tendency. The measure is representative of the data.

There are three widely used measures of central tendency.

1) Arithmetic Mean 2) Median and 3) Mode.

1] **Arithmetic Mean:** The arithmetical average of all observation in the given data is known as its arithmetic mean.

a] Mean of Ungrouped Frequency Distribution:

If x1, x2,………, xn are the values of a variable x, their mean is denoted by and is given as

$\overline{x} = \frac{x1 + x2 + \ldots\ldots + xn}{N} = \frac{1}{N}\sum xi$

where, N is the number of observations.

b] Mean of a Grouped Frequency Distribution:

There are three methods of computing the mean of a grouped data: A) Direct Method

B) Assumed Mean Method C) Step-deviation Method

i) Direct Method:

Step-1: In the table, make columns of values of $x_1, x_2, \ldots, x_n$ of a variable x with corresponding frequencies $f_1, f_2, \ldots, f_n$.

Step-2: Find class marks x_i.

Step-3: Make a column of f_ix_i.

Step-4: Find Σf_ix_i.

Step-5: Find Σf_i.

Step-6: Find $\overline{x} = \frac{\Sigma fixi}{\Sigma fi}$

ii) Assumed Mean Method

This method of solving the problem consists of the following steps:

Step-1: In the table, make columns of values of $x_1, x_2, \ldots, x_n$ of a variable x with corresponding frequencies $f_1, f_2, \ldots, f_n$.

Step-2: Find class marks x_i

Step-3: Take any value of class mark as assumed mean A. Generally take middle value as A.

Step-4: Make a column of $d_i = x_i - A$, d_i is called the deviation of x_i from A.

Step-5: Make a column of f_id_i.

Step-6: Find Σf_id_i.

Step-7: Find Σf_i.

Step-8: Find $\overline{d} = \frac{\Sigma fidi}{\Sigma fi}$

Step-9: Mean: $\overline{x} = A + \overline{d}$

iii) Step-deviation Method:

In the step-deviation we follow the 4 steps same as assumed mean method,

Step-5: Take G.C.D. of all values of d_i and we create a column for all u_i where, u_i =

Step-6: Make a column of f_iu_i. Find Σf_iu_i.

Step-7: Find Σf_i.

Step-8: Find $\overline{u} = \frac{\Sigma fiui}{\Sigma fi}$

Step-9: Mean: $\overline{x} = A + \overline{u}\ g$

2] **Median:** Median is the middle most term when the data is arranged in ascending or descending order.

a) Ungrouped data:

- If the number of terms, n is odd

  Median = $\left(\frac{n+1}{2}\right)$ term

- If the number of terms, n is even

  Median = $\frac{1}{2}\left[\left(\frac{n}{2}\right)\text{th term} + \left(\frac{n}{2}+1\right)\text{th term}\right]$

b) Grouped data:

Formula for finding the median of a grouped frequency distribution:

Median = $L + \frac{\frac{N}{2} - c.f.}{f} \times h$

where, L = lower boundary of the median class.

N = total frequency.

C.f. = cumulative frequency of the class preceding the median class.

f = frequency of the median class.

h = width of the median class.

3] **Mode:** The maximum occurred value among the observation is called mode.

a) Ungrouped data: In the raw data the observation repeating maximum number of times is mode.

b) Grouped data:

Mode of grouped frequency distribution is calculated using the following formula:

Mode = L + $\left[\frac{f_m - f_1}{2f_m - f_1 - f_2}\right] \times h$

where, L = lower boundary of the modal class.

f_m = frequency of the modal class [Modal class = A class which has maximum frequency.]

f_1 = frequency of the class preceding the modal class.

f_2 = frequency of the class succeeding the modal class.

h = width of the modal class.

Multiple Choice Questions

1. Find the mean of the daily income from the following frequency distribution:

Daily Income (in Rs)	100-150	150-200	200-250	250-300	300-350	350-400
Number of Workers	2	7	9	8	6	4

a) 250 Rs b) 254.17 Rs c) 127 Rs d) 235.20 Rs

2. The measurements (in mm) of the diameters of the head of the screws are given below:

Diameter (in mm)	33-35	36-38	39-41	42-44	45-47
Number of screw	17	19	23	21	27

Calculate mean diameter of the head of the screws.

a) 40.6 mm b) 40.2 mm c) 40 mm d) 38 mm

3. Find the mean marks from the frequency distribution given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of Students	3	5	7	10	12	15	12	6	2	8

a) 51 b) 57 c) 51.75 d) 54

4. Find the mean of rainfall (in cm) from the frequency distribution given below:

Rain fall (in cm)	36-40	40-44	44-48	48-52	52-56	56-60	60-64
Number of days	6	7	10	7	7	9	4

a) 49.6 cm b) 50 cm c) 52 cm d) 48 cm

5. Find the median:

Class	6-10	11-15	16-20	21-25	26-30
Frequency	20	30	50	40	10

a) 12 b) 15 c) 18 d) 20

6. Calculate the median:

Weight (in kg)	30-35	35-40	40-45	45-50	50-55	55-60
No. of Students	12	18	22	27	10	11

a) 45 kg b) 44.54 kg c) 42.59 kg d) 46.64 kg

7. Calculate the median:

Class	5-9	10-14	15-19	20-24	25-29	30-34	35-39

Frequency	3	12	29	47	19	12	5

a) 21.57　　b) 20.6　　c) 22.7　　d) 24.6

8. Calculate the mode:

No. of absent days (x)	0-10	10-20	20-30	30-40	40-50
No. of Students (F)	30	70	50	45	40

a) 15 absent days　　b) 18 absent days　　c) 15.2 absent days　　d) 16.67 absent days

9.

Class	35-40	40-45	45-50	50-55	55-60
Frequency	7	6	9	5	3

Calculate the mode.

a) 74.1　　b) 44.2　　c) 42.44　　d) 47.14

10. Find the mode:

Class	0-10	10-20	20-30	30-40
Frequency(F)	2	4	9	7

a) 27.14　　b) 26.2　　c) 25.5　　d) 22.2

11. The value of the mode in the following frequency distribution table is _____.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	20	32	14	14

a) 43　　b) 42　　c) 41　　d) 44

12. For a grouped frequency distribution, the relation between mean, mode and median is:

a) Mode = 3 median - 2 mean　　b) Mode = median - 2 mean

c) Mode = 2 median - mean　　d) 2 median - 3 mean

13. For a grouped frequency distribution mean = 24. 6 and mode = 29.1 then median =?

a) 26　　b) 26. 1　　c) 27　　d) 25.3

14. _____ is not a measure of central tendency.

a) mean deviation　　b) arithmetic mean　　c) median　　d) mode

15. Mean and median of a set of numbers 7 and 8 respectively. Then mode =?

a) 10　　b) 9　　c) 11　　d) 12

16. Mean of some scores is 10. If each score is multiplied by 4, the mean

a) 50　　b) 60　　c) 14　　d) 40

17. The mode of the given distribution is:

Weight (in kg)	40	43	46	49	52	55
No. of Children	5	8	16	9	7	3

a) 40　　b) 46　　c) 55　　d) None of these

18. The median of the following distribution is:

Class -interval	35-45	45-55	55-65	65-70
Frequency	8	12	20	10

a) 56.5　　b) 57.5　　c) 58.7　　d) 59

19. The standard deviation for the data:

a) 2.4　　b) 2.5　　c) 2.7　　d) 2.8

20. For a certain frequency distribution, values of mean and median are 62.6 and 62.5 respectively. Find the value of mode:

a) 6.23　　b) 62.3　　c) 63.5　　d) 62.5

21. Histogram consist of ____.

a) Sectors b) Rectangles c) Triangles d) Squares

22. The width of a rectangle in a histogram represents:
a) mid-values of the class b) class-interval c) frequency of the class d) number of classes

23. In a histogram:
a) The widths of all rectangles are equal
b) The lengths of all rectangles are equal
c) The lengths & width of all rectangles are equal
d) The length & width of each rectangle are in proportion

24. In a frequency polygon are used.
a) Mid points of classes and frequencies.
b) End points of classes and frequencies.
c) Upper boundaries of classes less than cumulative frequencies.
d) Lower boundaries of classes, greater than cumulative frequencies are used.

25. For drawing less than cumulative frequency
a) Upper boundaries of classes, cumulative frequencies.
b) Lower boundaries of classes, cumulative frequencies.
c) Mid-values of classes, cumulative frequencies.
d) Upper boundaries of classes, less than cumulative frequencies.

Answer Keys

1. b	2. a	3. c	4. a	5. c	6. b	7. a	8. d	9. d	10. a
11. d	12. a	13. b	14. a	15. a	16. d	17. b	18. b	19. d	20. b
21. b	22. b	23. a	24. a	25. d					

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2.

Diameter (in mm)	Class Marks (xi)	No. of Screw (fi)	fi × xi
33-35	34	17	578
36-38	37	19	703
39-41	40	23	920
42-44	43	21	903
45-47	46	27	1242
Total		∑ fi = 107	∑ fi × xi = 4346

$\sum fi \times xi = 4346$, $\sum fi = 107 = \bar{x} = \frac{\sum fi \times xi}{\sum fi} = \frac{4346}{107} = 40.6$ ∴ answer is (a).

7.

Class	Class Boundary	Frequency	C.F.
5-9	4.5-9.5	3	3
10-14	9.5-14.5	12	15
15-19	14.5-19.5	29	44
20-24	19.5-24.5 (Median Class)	47	91 $\frac{N}{2} = 63.55$
25-29	24.5-29.5	19	100
30-34	29.5-34.5	12	122
35-39	34.5-39.5	5	127

Total	----	$\sum$ fi = 127	----

N = 127. $\frac{N}{2} = \frac{127}{2} = 63.5$

19.5 - 24.5 is median class. L = 19.5, f = 47, c.f. = 44, h = 5

Median = L + $\frac{\frac{N}{2} - \text{c.f.}}{f} \times h$ = 19.5 + $\frac{63.5 - 44}{47}$ x 5 = 19.5 + 2.07 = 21.57. ∴ answer is (a).

11. Mode = L + $\left[\frac{f_m - f_1}{2f_m - f_1 - f_2}\right] \times h$

∴ Modal Class = 40 - 50 ∴ $f_1 = 20$, $f_2 = 14$, $f_m = 32$ ∴ h = 50 - 40 = 10

L = Lower boundary of the model class = 40

∴ Mode = 40 + $\left[\frac{32-20}{2 \times 32-20-14}\right]$ x 10 = 40 + $\frac{12}{30}$ × 10 = 40 + 4 = 44 ∴ answer is (d).

13. Mode = 3 median - 2 mean ∴ 29.1 = 3 median - 2 × 24.6

∴ 29.1 + 2 × 24.6 = 3 median ∴ 29.1 + 49.2 = 3 median ∴ $\frac{78.3}{3}$ = median

∴ 26.1 = median. ∴ answer is (b).

15. Mode = 3 median - 2 mean = 3 (8) - 2 (7) = 24 - 14 ∴ Mode = 10 ∴ answer is (a).

17. Clearly, 46 occurs most often. So, mode = 46. ∴ answer is (b).

18.

Class Interval	Frequency	Cumulative frequency
35-45	8	8
45-55	12	20
55-65	20	40
65-75	10	50

Here, N = 50. So, $\frac{N}{2} = 25$

25 lies in the class interval 55-65.

∴ $L_1 = 55$, $L_2 = 65$, N = 50, C.f. = 20 and f = 20

Median = $L_1 + \frac{(L_2 - L_1)}{f}$ x $\left(\frac{N}{2} - \text{c.f.}\right)$ = 55+ $\frac{65-55)}{20}$ x (25 × 20) = 57.5 ∴ answer is (b).

19. M = $\frac{7+9+11+13+15}{5} = \frac{55}{5} = 11$.

∴ $\sum\delta^2 = |7-11|^2 + |9-11|^2 + |11-11|^2 + |13-11|^2 + |15-11|^2 = 40$

∴ $\sigma = \sqrt{\frac{\sum\delta^2}{N}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2} = 2 \text{ x } 1.41 = 2.8$ ∴ answer is (d).

20. Mean - Mode = 3 (Mean - Median)

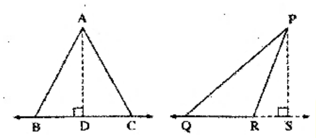
∴ 62.6 - mode = 3 (62.6 - 62.5) ∴ 62.6 - mode = 3 (0.1) ∴ 62.6 - mode = 0.3

∴ mode = 62.3 ∴ answer is (b).

9. Similarity

Important Facts and Formulae

I Properties of Area of Triangles:



The ratio of the areas of two triangles is equal to the ratio of the products of their bases and the corresponding heights.

In the figure,

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ i.e. $\frac{A_1}{A_2} = \frac{b_1 \times h_1}{b_2 \times h_2}$

where A, b, h denote area, base and height of triangle, respectively.

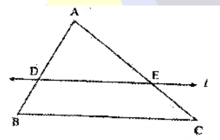
1] The ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases i.e. if $h_1 = h_2$ then $\frac{A_1}{A_2} = \frac{b_1}{b_2}$.

2] The ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights i.e. if $b_1 = b_2$ then $\frac{A_1}{A_2} = \frac{h_1}{h_2}$.

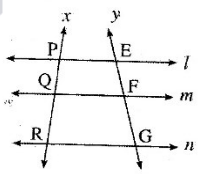
3] Areas of two triangles having equal bases and equal heights are equal. If $b_1 = b_2$, $h_1 = h_2$, then $A_1 = A_2$.

II. Basic Proportionality Theorem (B.P.T.):

If a line parallel to one side of a triangle intersects the other two sides in two distinct points, then the other two sides are divided in same ratio by it.



III. Property of intercepts made by three parallel lines:



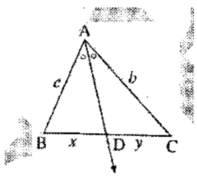
The ratio of the intercepts made on a transversal by three parallel line is equal to the ratio of corresponding intercepts made on any other transversal by the same parallel lines.

If line l || line m || line n

and line x and line y are transversal, then $\frac{PQ}{QR} = \frac{EF}{FG}$.

This property known as property of intercepts made by three parallel lines.

IV. Property of an angle bisector of a triangle:



In a triangle, the angle bisector divides the side opposite to the angle in the ratio of remaining sides

In ΔABC, ray AD is the bisector of $\angle A$.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$ i.e. $\frac{x}{y} = \frac{c}{b}$

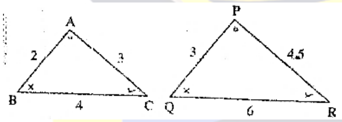
This property is known as the property of an angle bisector of a triangle.

V. Similarity of Triangles:

For a given one-to-one correspondence between the vertices of two triangles if

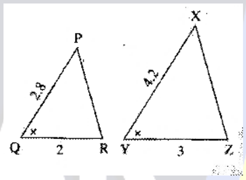
i) Their corresponding angles are congruent &

ii) Their Corresponding sides are proportional, then the correspondence is known as similarity & two triangles are said to be similar triangles.



1] SAS Test (side-angle-side test): For a given one-to-one correspondence between the vertices of two triangles, if two sides of one triangle are proportional to the corresponding sides of the other triangle and the angles included by them are congruent, then the two triangles are similar.

In the figure, under the correspondence,



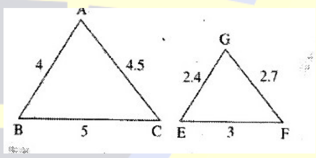
PQR $\leftrightarrow$ XYZ

$\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{2}{3}$ and $\angle Q \cong \angle Y$,

Then by SAS test, $\Delta PQR \sim \Delta XYZ$.

2] SSS Test: For a given one-to-one correspondence between the vertices of two triangles, if three sides of one triangle are proportional to the three corresponding sides of the other triangle, then the two triangles are similar.

In the figure, under the correspondence,



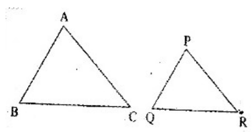
ABC $\leftrightarrow$ GEF

$\frac{AB}{GE} = \frac{BC}{EF} = \frac{AC}{GF} = \frac{5}{3}$

Then, by SSS test, $\Delta ABC \sim \Delta GEF$.

VI. Areas of Similar triangles:

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.



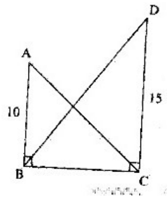
If $\Delta ABC \sim \Delta POR$, then $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

__Multiple Choice Questions__

1. In the figure, $\angle ABC = \angle DCB = 90^{\circ}$

AB = 10 and DC = 15.

Find the value of = $\frac{A(\Delta ABC)}{A(\Delta DCB)}$

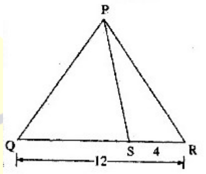


a) 3:5 b) 2:3 c) 5:2 d) 2:5

2. In the figure,

QR = 12, and SR = 4.

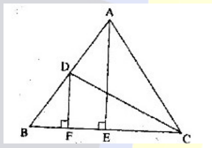
Find the value of = $\frac{A(\Delta PQS)}{A(\Delta PQR)}$



a) $\frac{1}{3}$ b) $\frac{8}{12}$ c) $\frac{2}{3}$ d) $\frac{2}{1}$

3. In the figure seg AE $\perp$ side BC

Find $\frac{A(\Delta AEC)}{A(\Delta DBF)}$

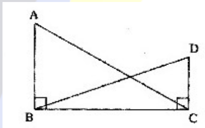


a) $\frac{AE}{DF}$ b) $\frac{BF}{FC}$ c) $\frac{EC \times AE}{BF \times DF}$ d) $\frac{AD \times DB}{BF \times EC}$

4. The ratio of the areas of two triangles with equal heights is 3 : 4 Base of smaller triangle is 15 cm. Find the corresponding base of larger triangle.

a) 20 cm b) 15 cm c) 4 cm d) 18 cm

5. In the figure $\frac{A(\Delta ABC)}{A(\Delta DBC)} = \frac{3}{2}$ (DC = 7 cm)



Find AB

a) 10.2 cm b) 10.5 cm c) 11.5 cm d) 9.5 cm

6. In a ΔABC, a line parallel to the side BC intersects the side AB and AC in the points M & N respectively. Such that AM = 8, MB = 12, AN - 6. Find NC.

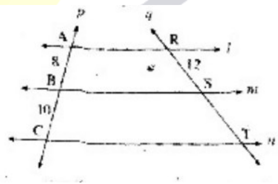
a) 9 b) 10 c) 12 d) 14

7. In the figure,

line l || line m || line n.

Lines P and q are the transversals.

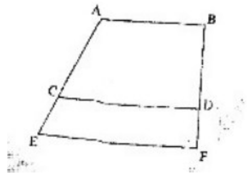
from the gives information, find ST.



$\frac{A_1}{A_2} = \frac{B_1}{B_2}$

a) 10 b) 15 c) 18 d) 20

8. In the figure,



seg AB|| seg CD|| seg EF,

AC = 15, BO = 10, DF = 6. Find AE.

a) 20 b) 24 c) 30 d) 18

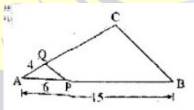
9. $\Delta PQR \sim \Delta XYZ$; PQ = 6, QR = 8, PR = 7, XY = 12 and $\angle Q = 75^o$. Find YZ.

a) 18 b) 16 c) 17 d) 15

10. $\Delta ABC \sim \Delta DEF$ and $\Delta ABC \sim \Delta PQR$. If DE = 8, AB = 7, PQ = 4, EF = 6, then find QR.

a) 2 b) 5 c) 3 d) 10

11. In the figure, $\Delta APO \sim \Delta ABC$;

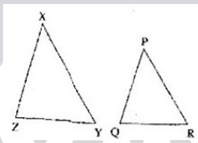


AP = 6, AB = 15, AQ = 4.

Find AC

a) 8 b) 7 c) 12 d) 10

12. In the figure, $\frac{PQ}{XZ} = \frac{2}{3}$



QR = 4, ZY = 6, PR = 6, XY = 9,

Are the ΔPQR and ΔXYZ similar? If so state the test.

a) SSS test b) SAS test c) AA test d) AAA test

13. In the trapezium ABCD, side AB||side DC. Diagonals AC and BD intersect each other at O. If AB = 15, CD = 10. OA = 9, find OC.

a) 7 b) 3 c) 6 d) 5

14. The sides of the smaller triangle out of the two similar triangles are 4, 5, and 6. If the perimeter of the larger triangle is 90, then what are the lengths of the sides of the larger triangle respectively.

a) 24, 30 & 36 respectively b) 20, 22 and 28 respectively

c) 18, 24 & 38 respectively d) 15, 17 & 20 respectively

15. $\Delta ABC \sim \Delta MNP$ and BC: NP 3: 4. Find A(ΔABC): (ΔMNP).

a) 6:15 b) 9:16 c) 9:5 d) 4:7

16. $\Delta LMN \sim \Delta RST$ and A (ΔLMN) = 100 sq. cm A(ΔRST) = 144 sq cm. LM = 5 cm, Find RS.

a) 5 cm b) 6 cm c) 8 cm d) 12 cm

17. Areas of two similar triangles are 225 cm^2 and 81 cm^2. If one side of the smaller triangle is 12 cm, then find the corresponding side of the larger triangle

a) 20 cm b) 15 cm c) 14 cm d) 25 cm

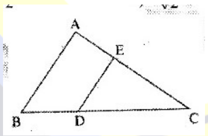
18. $\Delta PQR \sim \Delta PMN$ and 9 A(ΔPQR) = 16 A(ΔPMN). Find $\frac{QR}{MN}$

a) $\frac{4}{3}$ b) $\frac{3}{5}$ c) $\frac{5}{7}$ d) $\frac{3}{7}$

19. The side of an equilateral triangle is 8 cm. Find the side of equilateral triangle whose area is twice the area of the first.

a) $2\sqrt{2}$ b) $8\sqrt{2}$ c) $5\sqrt{2}$ d) $6\sqrt{2}$

20. In the figure,



seg DE || side AB.

DC = 2BD; A(ΔCBE) = 20 cm^2

Find A (□ABDE)

a) 25 cm^2 b) 30 cm^2 c) 35 cm^2 d) 20 cm^2

21. $\Delta ABC \sim \Delta PQR$. A($\Delta$ABC) = 16 cm^2 and A($\Delta$PQR) = 25 cm^2. Find $\frac{AB}{PQ}$

a) $\frac{4}{5}$ b) $\frac{2}{3}$ c) $\frac{5}{7}$ d) $\frac{7}{9}$

22. ΔABC ~ PQR, AB : PQ = 8 : 6. If A (Bigger triangle) = 48 cm^2, then A (smaller triangle) = …..cm^2

a) 10.66 b) 27 c) 36 d) 64

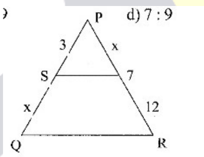
23. The distance between two places A and B is 175 km. In a map it is shown as 2.5 cm. In the same map d (C, A) = 3.2 cm. The actual d (C, A) is …… km.

a) 224 b) 455 c) 916 d) 1575

24. The corresponding sides of two similar triangles are 4 cm and 6 cm. Then the ratio of their area is……

a) 2:3 b) 3:5 c) 4:9 d) 7:9

25. ΔPQR, ST || QR, then x = ……



a) 6 b) 9 c) 12 d) 5.4

26. If ABC and DEF are similar triangles in which are similar triangles in which $\angle A = 47°$ and $\angle E = 83°$, then $\angle C$ is:
a) 50° b) 60° c) 70° d) 80°

27. If ΔABC and ΔDEF are so related that $\frac{AB}{FD} = \frac{AB}{DE} = \frac{CA}{EF}$ then which of the following is true?
a) $\angle A = \angle F$ and $\angle B = \angle D$ b) $\angle C = \angle F$ and $\angle A = \angle D$
c) $\angle B = \angle F$ and $\angle C = \angle D$ d) $\angle A = \angle E$ and $\angle B = \angle D$

28. The ratio of areas of two triangles having equal height is 2:3. If the base of the smaller triangle is 8 cm, the corresponding base of the remaining triangle is:
a) 8 cm b) 10 cm c) 11 cm d) 12 cm

29. The ratio of the corresponding sides of two similar triangles is 1:3. The ratio of their corresponding heights is:
a) 1:3 b) 3:1 c) 1:9 d) 9:1

30. The areas of two similar triangles are 49 cm2 and 64 cm2 respectively. The ratio of their corresponding sides is:

a) 49:64 b) 7:8 c) 64:49 d) None

Answer Keys

1. b	2. b	3. c	4. a	5. b	6. a	7. b	8. b	9. b	10. c
11. d	12. b	13. c	14. a	15. b	16. b	17. a	18. a	19. b	20. a
21. a	22. b	23. a	24. c	25. a	26. a	27. a	28. d	29. a	30. b

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3. In the ΔAEC, the base is EC and the height is AE.

In ΔDBF, the base is BF and height is DF.

$\therefore$ their areas are proportional to the products of the base and height. $\therefore$ Answer is (c).

6. In ΔABC, line MN || side BC $\therefore \frac{AM}{MB} = \frac{AN}{NC}$ (BPT)

$\therefore \frac{8}{12} = \frac{6}{NC}$ $\therefore$ 8NC = 72 $\therefore$ NC = 9 $\therefore$ Answer is (a).

7. Line l || line m|| line n

$\therefore$ by the property of intercepts made by three parallel lines,

$\therefore \frac{AB}{BC} = \frac{RS}{ST}$ $\therefore \frac{8}{12} = \frac{12}{ST}$ $\therefore$ 8ST = 120 $\therefore$ ST = 15

$\therefore$ Answer is (b).

12. In ΔAOB and ΔCOD, $\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle DAB \cong \angle OCD$ [alternate angles and AB || DC & AC transversal]

$\therefore \Delta AOB \sim \Delta COD$ .... (A - A test)

$\therefore \frac{OA}{OC} = \frac{AB}{CD}$ $\therefore \frac{9}{OC} = \frac{15}{10}$ $\therefore$ 15OC = 90 $\therefore$ OC = 6 $\therefore$ Answer is (c).

14. Let the lengths corresponding of side 4, 5 and 6 of smaller triangle be x, y, z respectively.

$\frac{X}{4} = \frac{Y}{5} = \frac{Z}{6}$ $\therefore \frac{x+y+z}{4+5+6} = \frac{x+y+z}{15}$ [theorem on equal ratios]

But perimeter of larger triangle = 90.

x + y + z = 90 $\therefore \frac{x+y+z}{15} = \frac{90}{15} = 6$ $\therefore \frac{x}{4} = 6, \frac{y}{5} = 6, \frac{z}{6} = 6$

$\therefore$ x = 24, y = 30 & z = 36 $\therefore$ Answer is (a).

17. Let $\angle ABC$ larger triangle; $\angle PQR$ smaller triangle

$\Delta ABC \sim \Delta PQR$ (given)

Let, the length of PQ of smaller triangle 12 cm.

By the theorem on areas of similar triangles,

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ $\therefore \frac{225}{81} = \frac{AB^2}{(12)^2}$ $\therefore \frac{15}{9} = \frac{AB}{12}$ $\therefore$ 9 × AB = 12 x 15 $\therefore$ AB = 20

$\therefore$ Answer is (a).

19. Let ΔABC is equilateral triangle having side 8 cm

ΔPQR is another equilateral triangle,

$A(\Delta PQR) = 2A(\Delta ABC)$

$\Delta ABC \sim \Delta PQR$ (equilateral triangles are always similar)

By the theorem on areas of similar triangles

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ $\therefore \frac{A(\Delta ABC)}{2A(\Delta ABC)} = \frac{8^2}{PQ^2}$ $\therefore \frac{1}{2} = \frac{64}{PQ^2}$

$PQ^2 = 64 \times 2$, $PQ = \sqrt{64 \times 2} = 8\sqrt{2}$ cm $\therefore$ Answer is (b).

22. The ratio of the areas of two similar triangles equals the ratio of the squares of their corresponding sides.

Let ΔABC be the bigger triangle and ΔPQR be the smaller triangle.

Let $A(\Delta PQR) = x$, then $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{8^2}{6^2} = \frac{64}{36}$ $\therefore \frac{64}{36} = \frac{48}{x}$

$\therefore x = 48 \times \frac{36}{64} = 27cm^2$ $\therefore$ Answer is (c).

24. $\frac{A(\text{Smaller } \Delta)}{A(\text{Bigger } \Delta)} = \frac{4^2}{6^2} = \frac{16}{36} = \frac{4}{9}$ = 4:9 ∴ Answer is (c).

25. ST || QR ∴ $\frac{3}{x} = \frac{x}{12}$ ∴ $x^2 = 3 \times 12$ ∴ x = 6. ∴ Answer is (a).

26. ∠A = ∠D, ∠B = ∠E and ∠C = ∠F ∴ ∠D = ∠A = 47°,
∠E = ∠B = 83° ∴ ∠C = 180° - (∠A + ∠B) = 180° - (47° + 83°) = 50°
∴ Answer is (a).

27. Clearly, A ⟷ F, B ⟷ D and C ⟷ E
∴ ∠A = ∠F and ∠B = ∠D ∴ Answer is (a).

28. The ratio of the areas of two triangle of equal heights is equal to the ratio of their corresponding bases.
∴ Let the base of the remaining triangle be x cm.
Then $\frac{A(\text{Smaller } \Delta)}{A(\text{Remaining } \Delta)} = \frac{2}{3} = \frac{8}{x}$ ∴ $\frac{2}{3} = \frac{8}{x}$ ∴ $x = 8 \times \frac{3}{2}$ ∴ x = 12 cm ∴ Answer is (d).

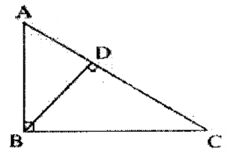
29. Ratio of height = Ratio of sides = 1:3 ∴ Answer is (a)

30. $\frac{(\text{Side of first } \Delta)^2}{(\text{Corresponding side of 2nd } \Delta)^2} = \frac{49}{64}$ ⇒ Ratio of corresponding sides = 7: 8
∴ Answer is (b).

10. Theorem of Pythagoras

Important Facts and Formulae

I. In a right angled triangle, if the perpendicular is drawn from the vertex of the right angle to the hypotenuse then triangles on either side of perpendicular are similar to the original triangle and to each other



In ΔABC, $\angle B = 90°$,

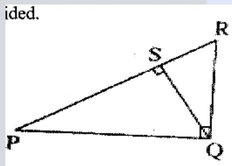
seg $BD \perp$ side AC

$\therefore$ i) $\Delta ADB \sim \Delta ABC$

ii) $\Delta BDC \sim \Delta ABC$

iii) $\Delta ADB \sim \Delta BDC$

II. In a right angled triangle, perpendicular segment to the hypotenuse from the opposite vertex is the geometric mean of the segments into which hypotenuse is divided.



In ΔPQR, $\angle Q = 90°$, seg $QS \perp$ side PR

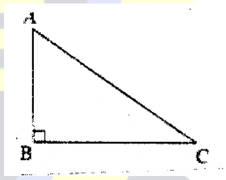
$\therefore QS^2 = PS \times RS$

$\therefore QS = \sqrt{PS \times RS}$

i.e. seg QS is geometric mean of seg PS and seg RS.

This is known as property of geometric mean.

III. **THEOREM OF PYTHAGORAS:**



In a right-angled triangle, the square of the hypotenuse

is equal to the sum of the squares of the remaining two sides

In ΔABC, $\angle ABC = 90°$.

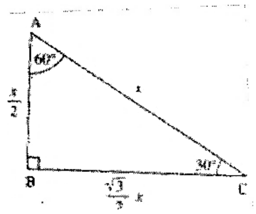
side AC is hypotenuse.

$\therefore$ By the theorem of Pythagoras,

$AC^2 = AB^2 + BC^2$

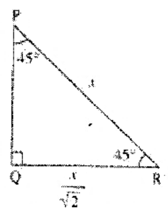
APPLICATIONS OF PYTHAGORAS' THEOREM:

1. **Theorem of 30°- 60°- 90° triangle:**



If the angles of triangles are 30°, 60°, 90° then the side opposite to 30° is half of hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse

2. **Theorem of 45°- 45°- 90° triangle:**



If the angles of a triangles are 45°, 45° & 90°, then each of the

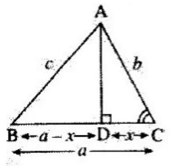
perpendicular sides is $\frac{1}{\sqrt{2}}$ times of the hypotenuse.

3. If in an acute angled $\triangle ABC$,

$\angle C$ is an acute angle and seg $AD \perp$ side BC & $DC = x$

then we have $c^2 = a^2 + b^2 - 2ax$

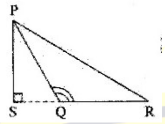
i.e. $AB^2 = AC^2 + BC^2 - 2BC.DC$



4. If in an obtuse-angled $\triangle PQR$, $\angle PQR = 90°$

If seg $PS \perp$ line QR and S-Q-R,

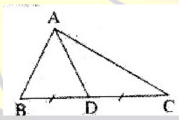
then $PR^2 = PQ^2 + QR^2 + 2QR.SQ$



5. THEOREM OF APPOLONIUS

If D is midpoint of side BC,

then $AB^2 + AC^2 = 2AD^2 + 2BD^2$



Multiple Choice Questions

1. In right angled triangle, two sides making right angle are 9 cm and 12 cm. Find the hypotenuse

a) 20 cm b) 10 cm c) 15 cm d) 25 cm

2. Find the diagonal of a square whose side is 20 cm.

a) 15 cm b) 20 c) 10 cm d) 7 cm

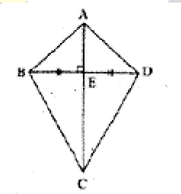
3. In a right angled triangle two side making right angle are 5 cm & 12 cm find the hypotenuse

a) 14 b) 13 c) 12 d) 15

4. In the Figure, diagonal AC is perpendicular bisector of diagonal BD.

BD=16 cm AB = 10 and BC = 17 cm.

Find the length of diagonal AC.



a) 20 cm b) 15 cm c) 21 cm d) 14 cm

5. In a right angled $\triangle ABC$, hypotenuse BC = 65 cm AB = 56 cm, Find AC

a) 35 cm b) 34 cm c) 15 cm d) 33 cm

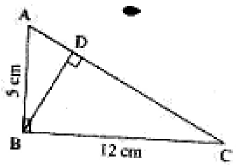
6. The side of a square is 8 cm. Find the length of its diagonal

a) 6 cm b) 8 cm c) 7 cm d) 10 cm

7. The length of rectangle is 35 m and breadth is 12 m. Find the length of its diagonal.

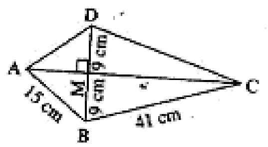
a) 35 m b) 15 m c) 37 m d) 41 m

8. Observe the figure and find AC



a) 13 cm c) 15 cm c) 20 cm d) 18 cm

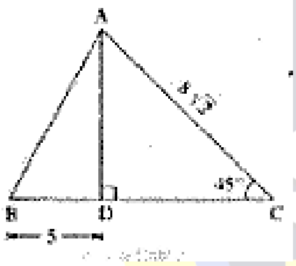
9. In the figure, diagonal AC bisects diagonal BD at right angles. AB = 15 cm, BC = 41 cm, BD = 18 cm, find AC



a) 50 cm b) 15 cm c) 52 cm d) 58 cm

10. In the figure, seg AD ⊥ side BC.

$\angle C = 45°$, AC = $8\sqrt{2}$, BD = 5. find AD



a) 7 cm b) 8 cm c) 6 cm d) 5 cm

11. □ AMRT is a parallelogram. $\angle m = 60°$.seg AP ⊥ side MR. AM = 8 cm., MR = 12 cm.

Find seg AP

a) $2\sqrt{2}$ b) $4\sqrt{3}$ c) $5\sqrt{2}$ d) $7\sqrt{3}$

12. In ΔABC, $\angle ABC = 90°$, AB = 12, BC = 16. seg BP is median. Find BP.

a) 10 b) 12 c) 14 d) 15

13. Adjacent sides of parallelogram are 11 cm and 17 cm. Its one diagonal is 26 cm. find its other diagonal.

a) 10 cm b) 11 cm c) 12 cm d) 13 cm

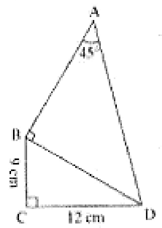
14. In ΔPQM, $\angle Q = 90°$, seg QR ⊥ side PM. QR = 12, PR = 9 Find area of ΔPQM.

a) 120 sq. unit b) 135 sq. unit c) 150 sq. unit d) 110 sq. unit

15. In the figure, $\angle A = 45°$,

CD = 12 cm, BC = 9 cm.

$\angle ABD = \angle BCD = 90°$ and find A(ΔABCD)



a) 50 cm^2 b) 54 cm^2 c) 40 cm^2 d) 55 cm^2

16. In ΔPQR, $\angle Q = 90°$. seg QM $\perp$ side PR. PM=9, MR = 25 find QM.

a) 12 b) 16 c) 15 d) 19

17. Area of rectangle is 192 sq. cm and its length is 16 cm. Find the diagonal of the rectangle

a) 18 cm b) 20 cm c) 19 cm d) 15 cm

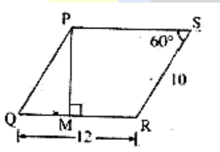
18. In ΔABC, $AB^2 + AC^2 = 122$, BC = 10. Find the length of the median on side BC.

a) 5 b) 6 c) 8 d) 10

19. In right-angled triangle, hypotenuse is 61 cm and one side is 11 cm. Find its other side.

a) 60 cm b) 58 cm c) 70 cm d) 55 cm

20. □PQRS is a parallelogram. seg PM $\perp$ side QR from the information given in the figure find A(□PQRS)



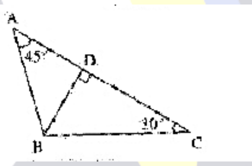
a) $20\sqrt{3}$ sq. units b) $30\sqrt{2}$ sq. units c) $60\sqrt{3}$ sq. units d) $50\sqrt{3}$ sq. units

21. seg AM is the median of triangle ABC. If BC = 16 cm, $AB^2 + AC^2 = 200$ cm^2 Find AM.

a) 5 cm b) 6 cm c) 7 cm d) 10 cm

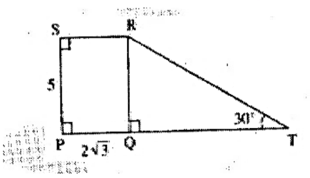
22. In the figure, seg BD $\perp$ side AC.

$\angle C = 30^0$, $\angle A = 45^0$. BD = 20 cm. Find BC.



a) 40 cm b) 30 cm c) 36 cm d) 42 cm

23. Observe the figure and find RT.



a) 8 cm b) 10 cm c) 20 cm d) 15 cm

Answer Keys

1. c	2. b	3. b	4. c	5. d	6. b	7. c	8. a	9. c	10. b
11. b	12. a	13. c	14. c	15. b	16. c	17. b	18. b	19. a	20. c
21. b	22. a	23. b							

HINTS AND SOLUTIONS

4. Diagonal AC is perpendicular bisector of diagonal BD

$\therefore \triangle AEB$ is right-angled triangle and $BE = \frac{1}{2} BD = \frac{1}{2} \times 16$ $\therefore BE = 8cm$

In right-angled $\triangle AEB$, by Pythagoras theorem,

$AB^2 = AE^2 + BE^2$

$\therefore AE^2 = AB^2 - BE^2$ $\therefore AE^2 = 10^2 - 8^2$ $\therefore AE^2 = 100 - 64$ $\therefore AE = 6$ cm

Similarly in right angled $\triangle BEC$ by pythagoras theorem,

$BC^2 = BE^2 + CE^2$ $\therefore CE^2 = BC^2 - BE^2 = 17^2 - 8^2 = 289 - 64 = 225$ $\therefore CE = 15$ cm

$AC = AE + CE$ $\therefore (6 + 15)$ cm $= 21$ cm $\therefore$ Answer is (c).

10. In $\triangle ADC$, $\angle C = 45^\circ$, $\angle ADC = 90^\circ$ $\therefore \angle CAD = 45^\circ$ (remaining angle of $\triangle ADC$)

$\therefore \triangle ADC$ is $45^\circ - 45^\circ - 90^\circ$ triangle

$\therefore AD = DC = \frac{1}{\sqrt{2}} AC = \frac{1}{\sqrt{2}} \times 8\sqrt{2} = 8$ cm $\therefore$ Answer is (b).

14. In $\triangle PQM$, $\angle Q = 90^\circ$. seg $QR \perp$ side PM.

$\therefore$ seg QR is the geometric mean of seg PR & seg RM.

$\therefore QR^2 = PR \times RM$ $\therefore (12)^2 = 9 \times RM$ $\therefore RM = \frac{12 \times 12}{9}$ $\therefore RM = 16$

$PM = PR + RM$ $\therefore PM = 9 + 16 = 25$

$A(\triangle PQM) = \frac{1}{2} \times PM \times QR = \frac{1}{2} \times 25 \times 12 = 150$ sq. unit

$\therefore$ Answer is (c).

18. Let seg AM be median on BC. Then BM = CM

$\therefore \frac{1}{2} BC = \frac{1}{2} \times 10$ $\therefore MB = CM = 5$.

By Appolonius theorem, $AB^2 + AC^2 = 2AM^2 + 2BM^2$ $\therefore 122 = 2AM^2 + 2(5)^2$

$\therefore 122 = 2AM^2 + 50$ $\therefore 2AM^2 = 72$ $\therefore AM^2 = 36$ $\therefore AM = 6$

$\therefore$ Answer is (b).

19. Let $\triangle ABC$ be right angled triangle $\angle B = 90^\circ$

Hypotenuse = AC = 61 cm, side BC = 11 cm

By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$\therefore AB^2 = AC^2 - BC^2 = (61)^2 - (11)^2 = (61 - 11)(61 + 11) = 50 \times 72 = 2 \times 25 \times 2 \times 36$

$\therefore AB = 2 \times 5 \times 6 = 60$ cm $\therefore$ Answer is (a).

20. Opposite angles of parallelogram are congruent.

$\therefore \angle Q = \angle S = 60^\circ$ ....(i) opposite sides of a parallelogram are equal

∴ $PQ = SR = 10$ .....(ii)

In right angled, ΔPQM, $\angle Q = 60^\circ$, $\angle PMQ = 90^\circ$, $\angle QPM = 30^\circ$, ΔPQM is $30^\circ - 60^\circ - 90^\circ$ triangle

∴ $PM = \frac{\sqrt{3}}{2} PQ = \frac{\sqrt{3}}{2} \times 10$ ∴ $PM = 5\sqrt{3}$

Area of parallelogram = base × height

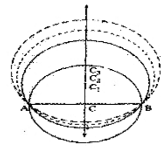
∴ $A[PQRS] = QR \times PM = 12 \times 5\sqrt{3} = 60\sqrt{3}$ sq. Units ∴ Answer is (c).

11. Circle : Tangent

Important Facts and Formulae

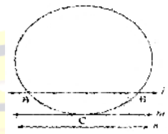
1 Observe the figure:

We can draw infinite number of circles passing through two given points A and B.



If three points are collinear, there is no circle passing through these points, because there are at the most two points common to a circle and a line.

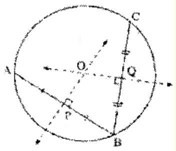
Observe the figure: (i) The circle and the line l have two points, A and B, common.



(ii) The circle and the line m have only one point, C common.

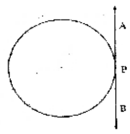
(iii) The circle and line n have no common point.

Theorem - 1: There is one and only one circle passing through given three non-collinear points.



Tangent and its properties:

Tangent line: A line in the plane of a circle which intersects the circle in one and only one point is called a tangent of the circle. the point of intersection is called the point of contact.

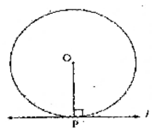


In the figure, the line AB is a tangent to the circle at the point P. Point P is the point of contact.

Theorem 2:

A tangent at any point of a circle is perpendicular to the radius through the point of contact.

In the figure, line l is the tangent to the circle at the point P. Seg OP is the radius through the point of contact P.



∴ line l ⊥ radius OP.

Theorem 3:

The line perpendicular to a radius at its outer end is a tangent to the circle.

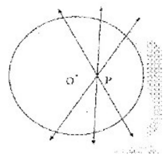
In the figure, line l is perpendicular to radius OP at its outer end P.

∴ line l is a tangent to the circle.

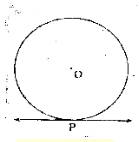
Number of Tangents to a Circle through a given point:

Given a point P in the plane of a circle with centre O:

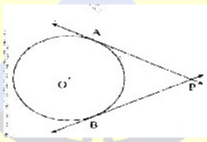
1] If the point P is inside the circle, then every line passing through the point P intersects the circle in two distinct points. Hence, none of them is a tangent to the circle.



2] If the point P is on the circle, then there is one and only one tangent to the circle passing through the point P.



3] If the point P is in the exterior of the circle, then two tangents can be drawn to the circle from the point P.



Theorem 4:

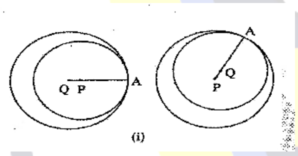
The length of the two tangent segments from an external point to a circle are equal.

In the figure given in (3) above, PA = PB.

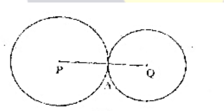
Touching Circles:

1. Tangent circles: Two coplanar circles are said to be touching circles or tangent circles, if they have one and only one point in common.

i) If two circles touch each other and one circle is in the interior of the other, the circles are internally touching circles.



ii) If two circles touch each other and one circle is in the exterior of the other, the circles are externally touching circles.



Theorem 5:

If two circles are touching circles, then the common point (i.e., the point of contact) lies on the line joining their centres.

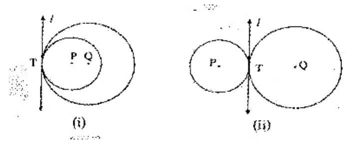
In the above figure (i), Q - P - A or P - Q - A.

In the figure (ii), P – A – Q.

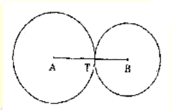
If two circles touch each other, then there is a line passing through their point of contact and tangent to both the circles. (The tangent is called the common tangent to the touching circles.)

Observe the following figures:

i) If two circles are touching internally, then both the circles are on the same side of the common tangent.



ii) If two circles are touching externally, then both the circles are on the opposite sides of the common tangent.

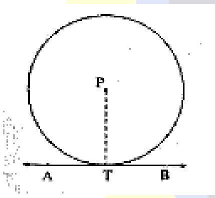


Multiple Choice Questions

1. Suppose points O, A, B, C, D, E are such that OA = 5 cm, OB = 6 cm, OC = 5 cm, OD = 4 cm, OE = 5 cm. Out of A, B, C, D, E, state which points lie on the same circle with centre O and radius 5 cm.

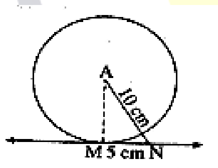
   a) A, B, C b) C, D, E c) A, C, E d) B, A, D

2. In the figure, P is the centre of the circle and line AB is the tangent to the circle at T. The radius of the circle is 6 cm. Find the distance of P from the line AB.



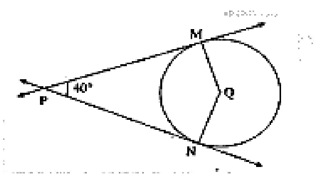
   a) 5 cm b) 6 cm c) 8 cm d) 10 cm

3. In the figure, A is the centre of circle. AN = 10 cm, Line NM is the tangent at M. Determine the radius of the circle, if MN = 5 cm.



   a) $2\sqrt{3}$ cm b) $5\sqrt{3}$ cm c) $6\sqrt{3}$ cm d) $4\sqrt{3}$ cm

4. In the figure, Q is the centre of the circle and PM & PN are tangent segments to the circle. If MPN = 40° find ∠MQN



   a) 130° b) 100° c) 140° d) 125°

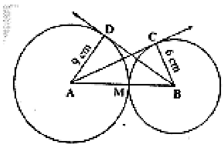
5. In the above example, if the radius of the circle is 7 cm and PM = 7cm determine the distance QR.

   a) $6\sqrt{2}$ cm b) $9\sqrt{2}$ cm c) $8\sqrt{2}$ cm d) $7\sqrt{2}$ cm

6. P is the centre of the circle. Line AB is the tangent to the circle at the point T. The radius of the circle is 5 cm. Find the distance of centre from line AB.

a) 6 cm b) 5 cm c) 8 cm d) 10 cm

7. In the figure, A & B are the centres of two circles touching each other at the point M. Line AC and line BD are tangents. If AD = 9 cm & BC = 6 cm, then find the lengths of seg AC & seg BD.

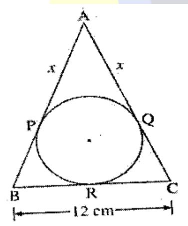


a) AC = $3\sqrt{21}$ cm; BD = 12 cm b) AC = $2\sqrt{21}$ cm; BD = 12 cm

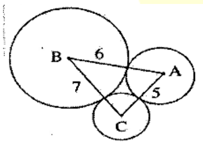
c) AC = $3\sqrt{21}$ cm; BD = 16 cm d) AC = $3\sqrt{23}$ cm; BD = 14 cm

8. In the figure ΔABC is an isosceles triangle with perimeter 44 cm. The base BC is of length 12 cm. sides AB and AC are congruent. A circle touches the sides as shown. Find the length of tangent segment from A to the circle

a) 10 cm b) 12 cm c) 8 cm d) 14 cm



9. In the given figure, A, B, C are the centre of there tangent circles. AB = 6 cm, BC = 7 cm, AC = 5 cm. Find the radius of each circle.



a) 2 cm, 5 cm, 7 cm b) 9 cm, 5cm, 4 cm c) 8 cm, 6 cm, 8 cm d) 2 cm, 4 cm, 3 cm

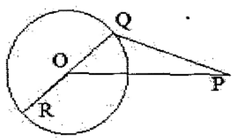
10. Find the length of tangent segment from a point which is at a distance of 5 cm from the centre of a circle of radius 3 cm.

a) 4 cm b) 6 cm c) 5 cm d) 3 cm

11. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is:

a) $\sqrt{7}$ cm b) $2\sqrt{7}$ cm c) 10 cm d) 5 cm

12. In the adjoining figure, PQ is a tangent from P to the circle and QOR is a diameter. If $\angle PQR = 130°$, then $\angle QPO$ is:



a) 40° b) 45^0 c) 50° d) 75°

13. If two equal circles touch each other externally, the common tangent divides the line of centres in the ratio

a) 1:1 b) 2:1 c) 1:2 d) 3:2

14. AC is tangent to a circle with centre O at the point A. ΔOAC is an isosceles triangle. ∠OCA is equal to:

a) 30° b) 45° c) 60° d) 90°

15. Two tangent at B and C from a point A to a circle with centre P are such that ∠BPC = 120° then ∠BAC = ?

a) 40° b) 60° c) 90° d) 50°

16. Two circle touch externally at P. A common tangent touches the circle at A and B. Then ∠APB = ?

a) 90° b) 60° c) 120° d) None of these

Answer Keys

1. c	2. b	3. b	4. c	5. d	6. b	7. a	8. a	9. d	10. a
11. c	12. a	13. a	14. b	15. b	16. a				

HINTS AND SOLUTIONS

1. Radii of same circle are equal. 5 cm

∴ OA = OC = OE

∴ The centre is equidistant from points A, C & E. ∴ Answer is (c).

2. Draw PT ∴ seg PT ⊥ line AB.

∴ PT is distance of the point P from the line AB.

PT = 6 cm (given r = 6 cm). ∴ Answer is (b)

3. Draw seg AM.

Line MN is tangent at point M and seg AM is radius AM ⊥ MN

∴ ∠AMN = 90°.

In right angled ΔAMN, by Pythagoras theorem,

$AN^2 = AM^2 + MN^2$

$AM^2 = AN^2 - NM^2$

$AM^2 = 10^2 - 5^2 = 75$ $AM = \sqrt{75} = 5\sqrt{3}$ ∴ Answer is (b)

4. seg PM and seg PN are tangents to the circle and seg QM and QN are the radius from the point of contact.

m∠PMQ = m∠PNQ = 90°.

The sum of measures of the angles of quadrilateral is 360°.

m∠P + m∠PMQ + m∠PNQ + m∠MQN = 360°

∴ 40° + 90° + 90° + m∠MQN = 360°

m∠MQN = 360° – 220°

$m\angle MQN = 140^{\circ}$.

$\therefore$ Answer is (c)

5. In the right-angled ΔPMQ, $\angle PMQ = 90^{\circ}$

$PQ^2 = PM^2 + MQ^2 = 7^2 + 7^2 = 98$ $\therefore PQ = \sqrt{98} = 7\sqrt{2}$cm $\therefore$ Answer is (d)

7. AM = AD = 9 cm ....(i)

similarly, BM = BC = 6 cm ... (ii)

AM + MB = AB (A-M-B)

$\therefore$ 9 + 6 = AB $\therefore$ AB = 15 cm.

By tangent-perpendicularity theorem, radius AD $\perp$ tangent BD at point Q

$\therefore$ ADB is right angled triangle

By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ $\therefore BD^2 = AB^2 - AD^2 = 15^2 - 9^2 = 225 - 81 = 144$ $\therefore$ BD = 12 cm.

Similarly in right ΔABC,

$AB^2 = BC^2 + AC^2$

$\therefore AC^2 = AB^2 - BC^2 = 225 - 36 = 189 = 9\times21$ $\therefore AC = 3\sqrt{21}$ cm $\therefore$ Answer is (a)

8. Perimeter of ΔABC = 44 cm.

AB + BC + AC = 44 cm.

$\therefore$ AB + AC = 44 - BC = 44 - 12 = 32 cm. AB = AC = 16 cm ..... (i) (Given)

The tangent segments to a circle from an external point are of equal lengths

$\therefore$ AP = AQ; BP = BR; CQ = CR

Let AP = AQ = x cm

BP = BR = (16 - x) cm .... (ii)

CQ = CR = (16 - x) cm .... (iii)

BR + RC = BC $\therefore$ 16 – x + 16 – x = 12 $\therefore$32 - 2x = 12 $\therefore$ 2x=20 $\therefore$ Answer is (a)

11. OP = 8 cm and OT = 6 cm

$\therefore PT = \sqrt{OP^2 + OT^2} = \sqrt{8^2 + 6^2} = 10$cm $\therefore$ Answer is (c).

12. $\angle QOP = (180^{\circ} - 130^{\circ}) = 50^{\circ}$ And, $\angle PQO = 90^{\circ}$

$\therefore \angle QPO = 180^{\circ} - (50^{\circ} + 90^{\circ}) = 40^{\circ}$ $\therefore$ Answer is (a).

13. Since the direct common tangent to two circle divides the line joining their centres externally in the ratio of their centres externally in the ratio of their radii. Here both the circles being of equal radii, this ratio is 1 : 1.

∴ Answer is (a).

14. Clearly, $OA \perp AC$. So, $\angle OAC = 90°$.

ΔOAC being isosceles, $OA = AC$. ∴ $\angle OCA = \angle COA$.

But $\angle OCA + \angle COA = 90°$ ∴ $\angle OCA = 45°$ ∴ Answer is (b).

15. Use tangent radius property and sum of the angles of a quadrilateral.

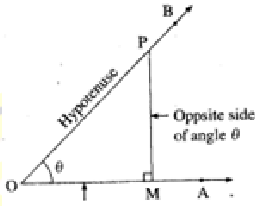
∴ Answer is (b).

Trigonometry
Important Facts and Formulae

I **Trigonometry:** Trigonometry is a branch of Mathematics that combines Arithmetic, Algebra and Geometry together. Study of Trigonometry is very useful in Engineering, Astronomy, Navigation, etc. Trigonometry deals with the measurement of the sides and the angles of a triangle. When some angles and sides of a triangle are given, we can obtain remaining angles and sides of the triangle using Trigonometry.

II. Trigonometric Ratios of Acute Angles:

Consider right-angled ΔPOM, ∠POM is an acute angle. It is denoted by θ. In ΔPOM, side OM is the adjacent side of angle θ.



Side PM is the opposite of angle θ. Side OP is the hypotenuse.

III. Trigonometrical Ratios:

1] $\sin\theta = \frac{\text{opposite side of angle}}{\text{hypotenuse}} = \frac{PM}{OP}$

2] $\cos\theta = \frac{\text{Adjacent side of angle}}{\text{hypotenuse}} = \frac{OM}{OP}$

3] $\tan\theta = \frac{\text{opposite side of angle}}{\text{adjacent side of angle}} = \frac{PM}{OM}$

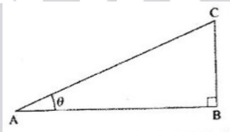
4] $\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{opposite side of angle}} = \frac{OP}{PM}$

5] $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side of angle}} = \frac{OP}{OM}$

6] $\cot\theta = \frac{\text{Adjacent side of angle}}{\text{opposite side of angle}} = \frac{OM}{PM}$

IV. Interrelation between the Trigonometric Ratios

In the figure, ΔABC is right-angled at B. ∠A = θ



1] $\sin\theta = \frac{BC}{AC}$ and $\text{cosec}\,\theta = \frac{AC}{BC}$

∴ $\sin\theta \times \text{cosec}\,\theta = \frac{BC}{AC} \times \frac{AC}{BC} = 1$.

∴ $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ and $\sin\theta = \frac{1}{\text{cosec}\,\theta}$

Similarly, we can write the following relations:

2] $\cos\theta \times \sec\theta = 1$, $\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta = \frac{1}{\sec\theta}$

3] $\tan\theta \times \cot\theta = 1$, $\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta = \frac{1}{\cot\theta}$

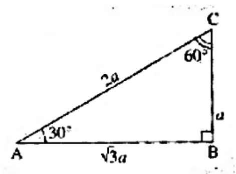
4] $\tan\theta = \frac{\sin\theta}{\cos\theta}$

5] $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Trigonometric Ratios of Angles 30°, 45° and 60°:

I) Trigonometric ratios of angle 30°:

In the adjoining figure, ΔABC is a right angled triangle.



m∠B = 90°, m∠A = 30° and m∠C = 60°.

∴ ΔABC is a 30°- 60°- 90° triangle

Let AC = 2a.

Then, by 30°- 60°- 90° triangle theorem, BC = a and AB = $\sqrt{3}$ a

$\therefore$ sin 30° = $\frac{BC}{AC} = \frac{1a}{2a} = \frac{1}{2}$

cos 30° = $\frac{AB}{AC} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$

tan 30° = $\frac{BC}{AB} = \frac{1a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$

cot 30° = $\frac{AB}{BC} = \frac{\sqrt{3}a}{1a} = \sqrt{3}$

cosec 30° = $\frac{AC}{BC} = \frac{2a}{1a} = 2$

sec 30° = $\frac{AC}{AB} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$

I) Trigonometric ratios of angle 60°:

Let us consider the trigonometric ratios of angle 60° in the above figure:

sin 60° = $\frac{AB}{AC} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$

cos 60° = $\frac{BC}{AC} = \frac{a}{2a} = \frac{1}{2}$

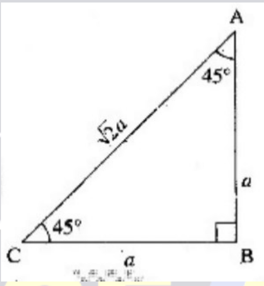
tan 60° = $\frac{AB}{BC} = \frac{\sqrt{3}a}{a} = \sqrt{3}$

cot 60° = $\frac{BC}{AB} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$

cosec 60° = $\frac{AC}{AB} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$

sec 60° = $\frac{AC}{BC} = \frac{2a}{a} = 2$

III) Trigonometric ratios of angle 45°:



In the adjoining figure, in ΔABC,

m$\angle$B = 90°, m$\angle$A = m $\angle$C = 45°. Let AB = BC = a.

Then, by pythagoras theorem, we get,

AC = $\sqrt{2}$a.

sin C = sin 45° = $\frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$.

cos C = cos 45° = $\frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$.

tan C = tan 45° = $\frac{AB}{BC} = \frac{a}{a}$ = 1.

cot C = cot 45° = $\frac{BC}{AB} = \frac{a}{a}$ = 1.

cosec C = tan 45° = $\frac{AC}{AB} = \frac{\sqrt{2}a}{a} = \sqrt{2}$

sec C = sec 45°= $\frac{AC}{BC} = \frac{\sqrt{2}a}{a} = \sqrt{2}$

IV) Trigonometric ratios of angles 0° and 90°

i) sin 0° = 0; cos 0° = 1; tan 0° = 0; sec 0° = 1; cosec 0° and cot 0° are not defined.

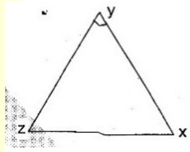
ii) sin 90° = 1; cos 90° = 0; cosec 0° = 1; cot 90° = 0; tan 90° and sec 90° are not defined.

The table for the values of trigonometric ratios of angles:

	0°	30°	45°	60°	90°
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

<u>Multiple Choice Questions</u>

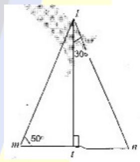
1. In the Figure, $\Delta xyz = 90^o$. Write trigonometric ratio for sec x and sec Z.



a) sec X = $\frac{ZX}{XY}$, sec Z = $\frac{ZX}{YZ}$ b) sec X = $\frac{XY}{ZY}$, sec Z = $\frac{YZ}{XY}$

c) sec X = $\frac{YZ}{ZX}$, sec Z = $\frac{YZ}{XY}$ d) sec X = $\frac{XY}{ZX}$, sec Z = $\frac{XY}{YZ}$

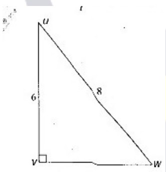
2. What is the value of tan 50°, sec 40°, sin 50° respectively from the given figure



a) $\frac{LT}{MT}, \frac{LM}{LT}, \frac{LT}{LM}$ b) $\frac{LT}{LT}, \frac{LT}{LM}, \frac{LT}{MT}$

c) $\frac{LT}{LM}, \frac{LT}{MT}, \frac{LM}{LT}$ d) $\frac{LN}{LT}, \frac{LM}{MT}, \frac{LT}{TN}$

3. What is the value of VW from the figure given alongside



a) $2\sqrt{7}$ b) $7\sqrt{2}$ c) $3\sqrt{2}$ d) $5\sqrt{2}$

4. If sinθ =$\frac{2}{7}$, find cosecθ.

a) $\frac{2}{7}$ b) $\frac{5}{2}$ c) $\frac{7}{2}$ d) $\frac{2}{5}$

5. If tanθ = $\frac{2}{5}$, find cot θ.

a) $\frac{2}{5}$ b) $\frac{5}{2}$ c) $\frac{7}{2}$ d) $\frac{9}{5}$

6. If sinθ = $\frac{\sqrt{2}}{3}$, cosθ = $\frac{1}{3}$, what is the value of tan θ.

a) $\frac{\sqrt{2}}{3}$ b) $\frac{3}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\sqrt{2}$

7. Find the value of 2 $\tan^2 45° + \cos^2 30° - \sin^2 60°$.

a) 4 b) 3 c) 2 d) 6

8. Find the value of $4\cot^2 45^\circ - \sec^2 60^\circ + \mathrm{cosec}^2 30^\circ + \cot 90^\circ$.

a) 4 b) 6 c) 8 d) 2

9. If $\cos(40^\circ + x) = \sin 30^\circ$, find the value of x.

a) 30° b) 40° c) 20° d) 60°

10. If $\tan y = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$, what is the value of y.

a) 40° b) 45° c) 60° d) 35°

11. Find the value of $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

a) 1 b) 2 c) 0 d) 3

12. Find the value of $\frac{\cos 80^\circ}{\sin 10^\circ} + 59^\circ\ \mathrm{cosec}\ 31^\circ$

a) 2 b) 4 c) 6 d) 8

13. $\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$

a) 2 b) 1 c) 0 d) 3

14. If $\tan 2A = \cot(A - 18^\circ)$, then find the value of A where (2A) and $(A - 18^\circ)$ are acute angles.

a) 30° b) 36° c) 42° d) 25°

15. If $\sin\theta = \frac{45}{33}$ find the value of $\mathrm{cosec}^2\theta - \cot^2\theta$

a) 2 b) 3 c) 1 d) 5

16. If 15 cot N = 8, Find the value of $\frac{1}{\sqrt{\sec^2 N - 1}}$

a) $\frac{7}{12}$ b) $\frac{8}{15}$ c) $\frac{2}{5}$ d) $\frac{9}{19}$

17. Find the value of A from $\sec 4A = \mathrm{cosec}(A - 20^\circ)$

a) 22° b) 44° c) 11° d) 33°

18. Find the value of A from $\tan 3A = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

a) 20° b) 10° c) 15° d) 25°

19. Find the value of 'x' if $\sin 2x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

a) 15° b) 20° c) 35° d) 40°

20. Find the value $\tan^2\theta + \cot^2\theta$ if $\tan\theta + \cot\theta = 2$

a) 5 b) 4 c) 2 d) 1

21. $\left(\frac{\sin 47°}{\cos 43°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2$ - 4 $\cos^2 45°$

a) 1 b) 2 c) 0 d) 3

22. Find the value of $\frac{\sin 50°}{\cos 40°} + \frac{\text{cosec } 40°}{\sec 50°}$ - 4 cos 50° cosec40°

a) 2 b) - 2 c) 3 d) 4

23. Find the value of 2 ($\cos^2 45°$ + $\tan^2 60°$) – 6 ($\sin^2 45°$ – $\tan^2 30°$)

a) 2 b) 4 c) 6 d) 3

24. Find the value of ‘x' if sin 2x = sin 60°cos 30° – cos 60°sin 30°.

a) 20° b) 15° c) 25° d) 30°

25. $\frac{1}{\text{cosec}^2 A.\ \cos A.\ \tan^2 A}$ =?

a) sin A b) cos A c) tan A d) cot A

26. $\frac{\sin\theta.\sec\theta.\text{cosec}\theta}{\tan\theta.\cos\theta.\cot\theta}$ = ?

a) $\sec^2\theta$ b) $\frac{1}{1} - \sin\theta$ c) $\frac{\sqrt{22}}{3}$ d) None of these

27. $\frac{\sec\theta}{\cos\theta} \times \frac{\text{cosec}\,\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta}$ × tan θ = ?

a) 2 + $\cot^2\theta$ + $\tan^2\theta$ b) 2 $\cot^2\theta$ + $\tan^2\theta$ c) 2 $\tan^2\theta$ + $\cot^2\theta$ d) 2 $\cot^2\theta.\tan^2\theta$

28. If sin x = $-\frac{1}{2}$ and x lies in 4th quadrant, then cos x is:

a) $\frac{-\sqrt{3}}{2}$ b) $\frac{-1}{\sqrt{2}}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{1}{2}$

29. If sin θ = $\frac{5}{13}$ then the values of tan θ and sec θ are respectively:

a) $\frac{12}{13}$ and $\frac{13}{12}$ b) $\frac{5}{12}$ and $\frac{13}{12}$ c) $\frac{13}{12}$ and $\frac{5}{12}$ d) $\frac{12}{5}$ and $\frac{13}{5}$

30. If sin θ and 90° < θ < 180°, then the value of the expression $\frac{2\sin\theta + \cos\theta}{3\cos\theta + 5\sin\theta}$ is:

a) $\frac{31}{85}$ b) $-\frac{1}{5}$ c) $\frac{1}{5}$ d) $-\frac{31}{85}$

31. If sin θ = $\frac{5}{13}$ and θ is acute, then the value of $\sqrt{(1+\tan\theta)(1-\tan\theta)}$ is:

a) $\frac{13}{5}$ b) $\frac{12}{15}$ c) $\frac{13}{12}$ d) None of these

32. If $\tan\theta = \frac{x}{y}$ then the value of $\left(\frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta}\right)$ equals:

a) $\frac{x^2+y^2}{x^2-y^2}$ b) $\frac{x^2-y^2}{x^2+y^2}$ c) $\frac{x}{\sqrt{x^2+y^2}}$ d) $\frac{y}{\sqrt{x^2+y^2}}$

33. If 5 tan θ = 4, the value of $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}\right)$ is:

a) $\frac{5}{3}$ b) $\frac{5}{6}$ c) $\frac{1}{6}$ d) $\frac{3}{10}$

34. If 16 cot x = 12, then $\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$ equals:

a) $\frac{1}{7}$ b) $\frac{3}{7}$ c) $\frac{2}{7}$ d) None of these

35. If tan θ = $\frac{3}{4}$ and 0° < θ < 90°, then the value of (sinθ cosθ) is:

a) $\frac{3}{5}$ b) $\frac{4}{5}$ c) $\frac{12}{25}$ d) None of these

36. If $\tan\theta = \frac{1}{\sqrt{7}}$, the value of $\left(\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}\right)$ is:

a) $\frac{5}{7}$ b) $\frac{3}{4}$ c) $\frac{3}{7}$ d) $\frac{1}{12}$

37. If tan θ = $\frac{4}{3}$ the value of $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}$ is:

a) $\frac{2}{3}$ b) $-\frac{1}{3}$ c) $\frac{1}{3}$ d) $\frac{3}{4}$

38. $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$ is equal to:

a) sec A + tan A b) $\sec^2 A + \tan^2 A$ c) $\sec^2 A \tan^2 A$ d) sec A tan A

39. $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$ is equal to:

a) cosec x + cot x b) cosec x - cot x c) cot x - tan x d) sec x - tan x

40. $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$ is equal to:

a) sec x + tan x b) sec x - tan x c) cosec x + cot x d) cosec x - cot x

Answer Keys

1. a	2. a	3. a	4. c	5. b	6. d	7. c	8. a	9. c	10. b
11. c	12. a	13. b	14. b	15. c	16. b	17. a	18. c	19. a	20. c
21. c	22. b	23. c	24. b	25. a	26. d	27. a	28. c	29. b	30. b
31. d	32. a	33. c	34. a	35. c	36. b	37. c	38. a	39. b	40. b

HINTS AND SOLUTIONS

5. tan θ . cot θ =1, $\frac{2}{5}$ × cot θ =1, cot θ = 1 × $\frac{5}{2}$ × $\frac{5}{2}$. ∴ Answer is (b).

8. cot 45° = 1, sec 60° = 2, cosec 30° = 2, cot 90° = 0

∴ $4\cot^2 45° - \sec^2 60° + \operatorname{cosec}^2 30° + \cot^2 90° = 4(1)^2 - (2)^2 + (2)^2 + 0 = 4 - 4 + 4 = 4$

∴ Answer is (a).

14. tan θ = cot (90 – θ)

∴ tan 2A = cot (90 – 2A) ......(1)

tan 2A = cot (A - 18°) ......(2)

from (2) & (1)

$A - 18° = 90° - 2A$ $\quad \therefore A + 2A = 90° + 18°$ $\quad \therefore 3A = 108°$ $\quad \therefore A = 36°$

∴ Answer is (b).

21. By the ratio of complementary angles,

$\cos\theta = \sin(90 - \theta)$ $\quad \therefore \cos 43° = \sin(90 - 43)° = \sin 47°$.

and $\cos 45° = \frac{1}{\sqrt{2}}$

$\therefore \left(\frac{\sin 47°}{\cos 43°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2 - 4\cos^2 45° = \left(\frac{\sin 47°}{\sin 47°}\right)^2 + \left(\frac{\sin 47°}{\sin 47°}\right)^2 - 4\left(\frac{1}{\sqrt{2}}\right)^2$

$= (1)^2 + (1)^2 - 4\left(\frac{1}{2}\right)$ $\quad = 1 + 1 - 2 = 2 - 2 = 0.$ $\quad$ ∴ Answer is (c).

25. $\frac{1}{\operatorname{cosec}^2 A \,.\, \cos A \,.\, \tan A} = \frac{1}{\operatorname{cosec}^2 A \,.\, \cos A \,.\, \frac{\sin A}{\cos A}} = \frac{1}{\operatorname{cosec}^2 A \,.\, \sin A} = \frac{\sin^2 A}{\sin A} = \sin A$

∴ Answer is (a)

26. $\frac{\sin\theta}{\tan\theta} \times \frac{\sec\theta}{\cot\theta} \times \frac{\operatorname{cosec}\theta}{\cos\theta} = \frac{(\sin\theta.\operatorname{cosec}\theta)\sec\theta}{(\tan\theta.\cot\theta)\cos\theta} = \frac{1 \times \sec\theta}{1 \times \cos\theta} = \sec^2\theta = \frac{1}{\cos^2 A} = \frac{1}{1 - \sin^2\theta}$

∴ Answer is (d).

27. $\frac{\sec\theta}{\cos\theta} \times \frac{\operatorname{cosec}\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} \times \tan\theta = \sec^2\theta.\operatorname{cosec}^2\theta.\cot\theta.\tan\theta = (1 + \tan^2\theta)(1 + \cot^2\theta) \times 1$

$= 1 + \cot^2\theta + \tan^2\theta + \tan^2\theta.\cot^2\theta \times 1 = 1 + \cot^2\theta + \tan^2\theta + 1 \times 1$

$= 2 + \cot^2\theta + \tan^2\theta$ $\quad$ ∴ Answer is (a).

28. In 4th quadrant, cos x is positive. $\quad \therefore \cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

∴ answer is (c).

29. $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$ and $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$ $\quad$ ∴ Answer is (b).

30. Since θ lies in the second quadrant, so cos θ is negative.

$\therefore \cos\theta = -\sqrt{1 - \sin^2\theta} = -\sqrt{\frac{1-64}{289}} = \frac{-15}{17}$ $\quad$ ∴ Answer is (b).

31. $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$. So $\tan\theta = \frac{5}{12}$

$\therefore \sqrt{(1 + \tan\theta)(1 - \tan\theta)} = \sqrt{1 - \tan^2\theta} = \sqrt{1 - \frac{25}{144}} = \frac{\sqrt{119}}{12}$ $\quad$ ∴ Answer is (a).

32. $\frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta} = \frac{x\tan\theta + y}{x\tan\theta - y}$ ....[Dividing Nr and Dr by cos 0]

$\left(\frac{x \times \frac{x}{y} + y}{x \times \frac{x}{y} - y}\right) = \left(\frac{x^2+y^2}{x^2-y^2}\right)$ ∴ Answer is (a).

33. Given $\tan\theta = \frac{4}{5}$ ∴ $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\tan\theta - 3}{5\tan\theta + 2}$ ....[Dividing Nr and Dr by cos]

$= \left(\frac{5\times\frac{4}{5}-3}{5\times\frac{4}{5}+2}\right) = \frac{1}{6}$ ∴ Answer is (c).

34. Given $\cot x = \frac{12}{16} = \frac{3}{4}$ ∴ $\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{1 - \cot x}{1 + \cot x}$ ....[Dividing Nr and Dr by cos x]

$= \left(\frac{1-\frac{3}{4}}{1+\frac{3}{4}}\right) = \left(\frac{1}{4} \times \frac{4}{7}\right) = \frac{1}{7}$ ∴ Answer is (a).

35. $\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$. So $\cos\theta = \frac{4}{5}$.

$\text{Cosec}\,\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\frac{16}{9}} = \frac{5}{3}$. So $\sin\theta = \frac{3}{5}$

∴ $\sin\theta\cos\theta = \left(\frac{3}{5} \times \frac{4}{5}\right) = \frac{12}{25}$ ∴ Answer is (c).

36. $\sec^2\theta = (1+\tan^2\theta) = \left(1+\frac{1}{7}\right) = \frac{8}{7}$; $= (1+\cot^2\theta) = (1+7) = 8$

$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \left(\frac{8-\frac{8}{7}}{8+\frac{8}{7}}\right) = \left(\frac{48}{7} \times \frac{7}{64}\right) = \frac{3}{4}$ ∴ Answer is (b).

37. $\text{Cosec}\,\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$. So $\sin\theta = \frac{4}{5}$

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}} = \sqrt{\frac{1}{5} \times \frac{5}{9}} = \frac{1}{3}$ ∴ Answer is (c).

38. $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} = \frac{1+\sin A}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

∴ Answer is (a).

39. $\sqrt{\frac{1-\cos x}{1+\cos x}} = \sqrt{\frac{1-\cos x}{1+\cos x} \times \frac{1+\cos x}{1+\cos x}} = \frac{1-\cos x}{\sqrt{1-\cos^2 x}} = \frac{1-\cos x}{\sin x} = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \text{cosec}\,x - \cot x$

∴ Answer is (b).

40. $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}} = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x}} = \frac{\sec x - \tan x}{\sqrt{\sec^2 x - \tan^2 x}} = \sec x - \tan x$

∴ Answer is (b).

13. Heights and Distances

Important Facts and Formulae

I. We already know that: In a right-angled Δ OAB, where $\angle$ BOA = θ

1] Sinθ = $\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{OB}$;

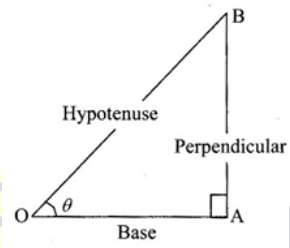
2] Cosθ = $\frac{\text{Base}}{\text{Hypotenuse}} = \frac{OA}{OB}$;

3] Tanθ = $\frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{OA}$

4] Cosecθ = $\frac{1}{\text{Sin }\theta} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{OB}{AB}$

5] Secθ = $\frac{1}{\text{Cos }\theta} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{OB}{OA}$

6] Cotθ = $\frac{1}{\text{Tan }\theta} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OA}{AB}$



II. Trigonometrical Identities:

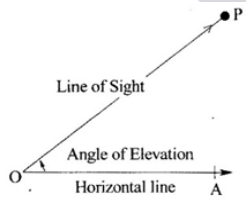
1] $\sin^2\theta + \cos^2\theta = 1$ 2] $1 + \tan^2\theta = \sec^2\theta$ 3] $1 + \cot^2\theta = \text{cosec}^2\theta$.

III. Values of T-ratios:

θ	0°	30° (π/6)	45° (π/4)	60° (π/3)	90° (π/2)
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cosecθ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cotθ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

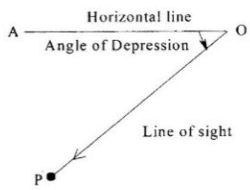
IV. Line of vision: If the observer is standing at the location 'A', looking at an object 'B' then the line AB is called line of vision.

V. Angle of Elevation: Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.



Angle of elevation of P from O = $\angle$AOP.

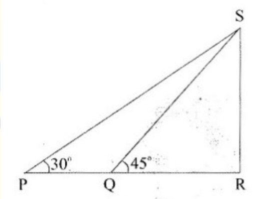
VI. Angle of Depression: Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



Angle of depression of P from O = $\angle$AOP.

Multiple Choice Questions

1. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is
a) 173 m b) 273 m c) 300 m d) 200 m
2. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of the elevation becomes 45°. What is the distance between the base of the tower and the point P?



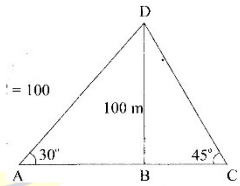
a) 9 units b) data inadequate c) 12 units d) 3 units
3. The angle of elevation of the sun, when the length of the shadow of a tree is equal to the height of the tree is ……
a) 45° b) 30° c) 60° d) None of these
4. The angle of elevation of the sun, when the length of the shadow of a tree is times the height of the tree,
a) 30° b) 45° c) 60° d) 90°
5. From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100m high, the distance point P from the foot of the tower is:
a) 149 m b) 156 m c) 173 m d) 200 m
6. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6m away from the wall. The length of the ladder is:
a) 2.3 m b) 4.6 m c) 7.8 m d) 9.2 m
7. An observer 1.6 m tall is 20 m away from a tower. The angle of elevation from his eye to the top of the m tall is the height of the tower is:
a) 21.6 m b) 23.2 m c) 24.72 m d) None of these
8. Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is:
a) 173 m b) 200 m c) 273 m d) 300 m
9. A man standing at a point P is Watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P?
a) 4 units b) 8 units c) 12 units d) Data inadequate
10. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top of the tower increases by 15o. The height of the tower is:
a) 17.3 m b) 21.9 m c) 27.3 m d) 30 m
11. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?
a) 32 kmph b) 36 kmph c) 38 kmph d) 40 kmph
12. On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, the distance between the objects is:
a) 63.5 m b) 76.9 m c) 86.7 m d) 90 m
13. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower?
a) 14 min. 35 sec. b) 15 min. 49 sec. c) 16 min. 23 sec. d) 18 min. 5 sec.

Answer Keys

1. b	2. b	3. a	4. a	5. c	6. d	7. a	8. c	9. d	10. c
11. a	12. a	13. c							

HINTS AND SOLUTIONS

1. Let BD be the lighthouse and A & C be the position of the ships.



Then, BD = 100 m, ∠BAD = 30°, ∠BCD = 45°

$\tan 30° = \frac{BD}{BA}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BA}$ $\Rightarrow BA = 100\sqrt{3}$

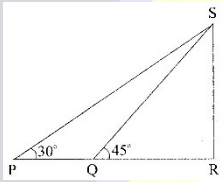
$\Rightarrow 1 = \frac{100}{BC}$ $\Rightarrow BC = 100$

Distance between the two ships = AC = BA + BC = $100\sqrt{3} + 100$

$= 100(\sqrt{3} + 1) = 100(1.73 + 1)$

$= 100 \times 2.73 = 273$ m

2. $\tan 45° = \frac{SR}{QR}$



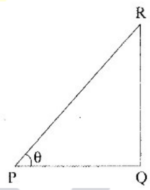
$\tan 30° = \frac{SR}{PR} = \frac{SR}{PQ + QR}$

Two equations and 3 variables.

Hence, we can not find the required value with the given data.

(Note that if one of the SR, PQ, QR is known, this becomes two equations and two variables and if that was the case, we could have found the required value.)

3. Consider the diagram shown alongside, where QR represents the tree and PQ represents its shadow.



We have, QR = PQ, Let ∠QPR = θ

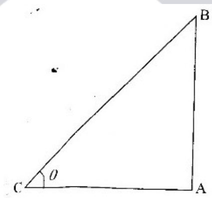
$\tan \Theta = \frac{QR}{PQ} = 1$ ...(Since QR = PQ)

$\Rightarrow \Theta = 45°$ i.e. required.

4. Let AB be the tree and AC be its shadow. Let ∠ACB = θ.

Then, $\frac{AC}{AB} = \sqrt{3}$ $\Rightarrow \cot \theta = \sqrt{3}$ $\Rightarrow \theta = 30°$.

5. Let AB be the tower. Then, ∠APB = 30° and AB = 100 m.



$\frac{AB}{AP} = \tan 30° = \frac{1}{\sqrt{3}}$ Ans: 173 m.

6. Let AB be the wall and BC be the ladder. Then, APB = 60° and AC = 4.6 m.

$\frac{AC}{BC} = \cos 60° = \frac{1}{2}$ Ans: 9.2 m.

7. Let AB be the observer and CD be the tower. Draw BE ⊥ CD.

$\frac{DE}{BE} = \tan 30° = \frac{1}{\sqrt{3}}$ Ans: 21.6 m.

8. Let AB be the lighthouse and C and D be the positions of the ships.

Then AB = 100 m, ∠ACB = 30° and ∠ADB = 45°.

$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}}$, $\frac{AB}{AD} = \tan 45° = 1.$ Ans: 273 m.

9. One of AB, AD and CD must have been given. So, data is inadequate.

10. Let AB be the tower and C and D be the points of observation.

Then, ∠ACB = 30° and ∠ADB = 45° and CD = 20 m.

Let AB = h. Then, $\frac{AB}{AC}$ = tan 30° = $\frac{1}{\sqrt{3}}$, $\frac{AB}{AD}$ = tan 45° = 1. Ans: 27.3 m.

11. Let AB be the tower and C and D be the two positions of the boats.

Then, ∠ACB = 45°, ∠ADB = 30° and AC = 60 m.

Let AE = h. Then, $\frac{AB}{AC}$ = tan 45° = 1, $\frac{AB}{AD}$ = tan 30° = $\frac{1}{\sqrt{3}}$. Ans: 32 km/hr.

12. Let AB be the tower and C and D be the objects.

Then AB = 150 m, ∠ACB = 45° and ∠ADB = 60°.

$\frac{AB}{AD}$ = tan 60° = $\sqrt{3}$ $\frac{AB}{AC}$ = tan 45° = 1 Ans: 33.5 m.

13. Let AB be the tower and C and D be the two positions of the car.

Then, ∠ACB = 45°, ∠ADB = 30° .

Let AB = h, CD = x and AC = y. $\frac{AB}{AC}$ = tan 45° = 1 $\frac{AB}{AD}$ = tan 30° = $\frac{1}{\sqrt{3}}$

Ans: 16 min. 23 sec.

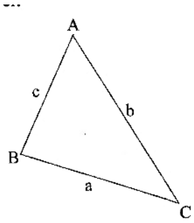
14. Area and Volume

Important Facts and Formulae

1) Triangle:

i) Area of triangle = $\frac{1}{2} \times b \times h$

ii) Heron's formula



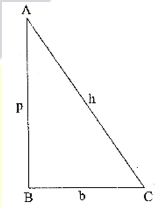
Area of triangle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

a + b + c = Perimeter

Thus, the sum of three sides of a triangle is called its perimeter.

S = $\frac{a+b+c}{2}$ is the semi-perimeter of the triangle.

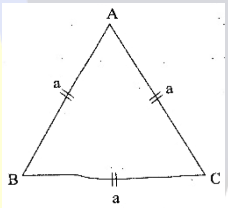
Pythagoras theorem: In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



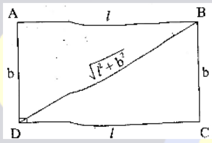
$AC^2 = AB^2 + BC^2$

$\therefore h^2 = p^2 + b^2$

iii) Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ $(\text{Side})^2$ = $\frac{\sqrt{3}}{4}$ $(a)^2$



2) Rectangle: For a rectangle of length l and breadth b, we have

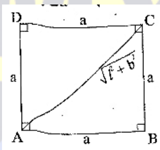


i) Perimeter = 2(l+b)

ii) Area = l x b

iii) Diagonal = $\sqrt{l^2 + b^2}$

3) Square: For a square, each of whose side is a, we have

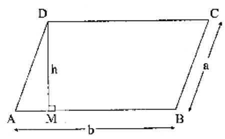


i) Perimeter = 4 × Length of side = 4l

ii) Area = $(\text{Side})^2 = a^2$

iii) Diagonal of square = $\sqrt{a^2 + a^2}$

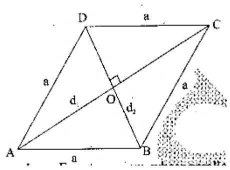
4) Parallelogram: For parallelogram, whose adjacent sides are b and a, we have



i) Perimeter = 2(a + b) = 2(Sum of adjacent sides)

ii) Area = Base x Height = AB x DM = bh

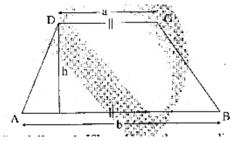
5) Rhombus: For a rhombus, whose diagonals are d_1, d_2, and each side is a, we have



i) Perimeter = 4 × Length of a side = 4a

ii) Area = $\frac{1}{2}$ (Product of its diagonals) = $\frac{1}{2} \times d_1 \times d_2$

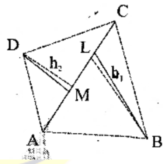
6) Trapezium: For a trapezium, whose parallel sides are a and b, and h is the distance between them, we have



Area = $\frac{1}{2}$ (Sum of parallel sides) × Distance between them

= $\frac{1}{2}$ (a + b) h

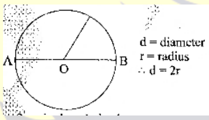
7) Quadrilateral: If h_1 and h_2 are the perpendicular distances on the distances on the diagonal AC of a quadrilateral ABCD from the vertices B and D, respectively, then



Area = $\frac{1}{2}$ (AC) ($h_1 + h_2$)

8) Circle:

i) Circumference of a circle:

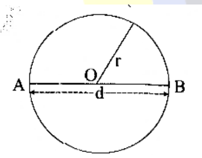


$\frac{\text{Circumference}}{\text{Diameter}}$ = π × Diameter = π × 2 × r = 2 π r

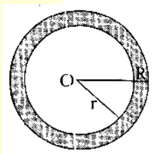
(where π = $\frac{22}{7}$ = 3.14)

ii) Area of a circle: It is the measurement of the surface enclosed by the circumference of the circle.

Area of a circle = πr^2 (where r = radius)



iii) A circular ring: It is an object bounded by the circumference of two concentric circles.



R = radius of the outer circle, r = radius of the inner circle

Area between circumferences of the two circles (shaded region)

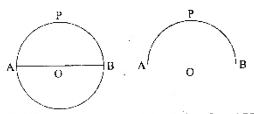
= Area of external circle - Area of internal circle

= $\pi R^2 - \pi r^2 = \pi (R^2 - r^2)$

Note: a) Distance travelled by a wheel in one revolution = Its circumference

b) Number of revolutions made by a wheel = $\frac{\text{Total distance travelled}}{\text{Circumference}}$

iv) Semicircle: Each diameter of a circle divides the circle into two congruent parts and each part is called a semicircle.

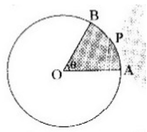


In the figure, AB is the diameter. Therefore, APB is a semicircle.

- Length of arc (circular part) = $\frac{1}{2}$ x (Circumference of the circle) = $\frac{1}{2}$ x $2\pi r = \pi r$
- Perimeter of the semicircle = Length of arc + Diameter AB = $\pi r + 2r$
- Area of the semicircle = $\frac{1}{2}$ × Area of circle = $\frac{1}{2} \times \pi r^2$

v) Sector of a circle: The part of a circle bounded by two radii and an arc is called a sector.

The shaded portion is a sector as it is bounded by two radii OA and OB and arc APB. If the angle between radii OA and OB is θ, then

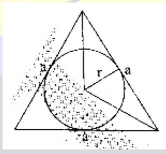


- Length of arc APB = $2\pi r \times \frac{\theta}{360°}$
- Perimeter of the sector = OA + OB + arc APB = $r + r + 2\pi r \times \frac{\theta}{360°}$
- Area of the sector = $\pi r^2 \times \frac{\theta}{360°}$

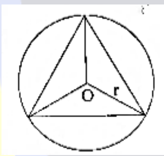
Note: If angle between the two radii is

- Less than 180°, the sector is called minor sector.
- More than 180°, the sector is called major sector.
- θ = 180°, the sector is one-fourths off the circle and is called gradrant.

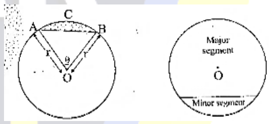
vi) Incircle: The radius of incircle of an equilateral triangle of side a = $\frac{a}{a\sqrt{3}}$



vii) Circumcircle: The radius of circumcircle of an equilateral triangle of side a = $\frac{a}{\sqrt{3}}$



viii) Segment of a circle: A segment of a circle is the region bounded by an arc and a chord. Including the arc and the chord, the segment containing the minor arc is called a minor segment and the segment containing the major arc is the major segment.

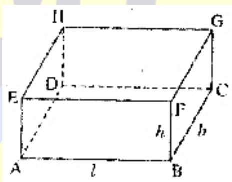


- Area of the minor segment ACBA = $\frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 sin\theta$
- Area of the major segment ADBA = πr^2 - (Area of the minor segment)
- Perimeter of the minor segment ACBA = $\frac{2\pi r\theta}{360°} - 2r sin\frac{\theta}{2}$

9) Cuboid:

1. Surface Area:

i)A cuboid has six rectangular faces.



ii) Opposite faces are parallel and congruent. They have equal area.

A(□ABCD) = A (□EFGH) = l x b = lb

A(□ADHE) = A (□BCGE) = b x h = bh

A(□ABFE) = A (□DCGH) = l × h = lh

a. The total surface area of a cuboid (S_t) = 2lb + 2bh + 2lh = 2 (lb + bh + lh)

b. The faces ADHE, BCGH, ABFE and DCGH are vertical faces.

Surface area of vertical faces

= bh + bh + lh + lh = 2bh + 2lh

= h(2b + 2l) = height × perimeter of the base

c. The faces ABCD and EFGH are horizontal faces.

Surface area of horizontal faces = lb + lb = 2lb.

2. Volume: Volume of a cuboid = l x b x h

10) Cube: A cube has six faces which are congruent squares.

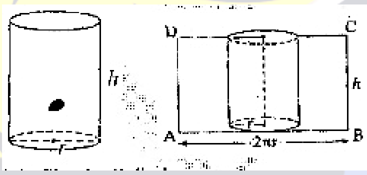
1. Total surface area of a cube = $6l^2$

2. Volume of a cube = l^3

11) Right Circular Cylinder:

1. Surface area of a cylinder:

i) Curved surface area of a cylinder (S_c)



S_c = Area of □ABCD = AB × BC = $2\pi r \times h$

∴ $S_c = 2\pi rh$

Right Circular Cylinder

ii) Total surface area of a cylinder (S)

$S_t = S_c$ + area of circular faces

$= 2\pi rh + 2\pi r^2$

∴ $S_t = 2\pi r\,(h + r)$

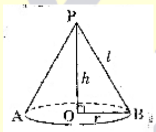
2. Volume of a cylinder:

Volume of a cylinder (V) = area of the base × height = $\pi r^2 \times h$

∴ $V = \pi r^2 h$

12) Cone (Right Circular Cone):

1. For a cone:



$l^2 = h^2 + r^2$,

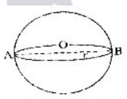
where l is the slant height, h is the vertical height and r is the radius.

2. Curved Surface area of a cone = $S_c = 2\pi rl$

3. Total surface area (S_t) of a cone = $\pi r^2 + \pi rl = \pi r(r + l)$

4. Volume of a cone = $\frac{1}{3} r^2 h$

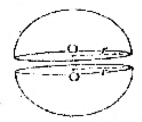
12) Sphere:



1. Surface area of a sphere = $4\pi r^2$

2. Volume of a sphere = $\frac{4}{3}\pi r^3$

14) Hemisphere:



1. Curved surface area of a hemisphere = $S_c = 2\pi r^2$

2. Total surface area of a closed hemisphere = $S_t = 3\pi r^2$

3. Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Multiple Choice Questions

1. If the area of a triangle is 1176 cm^2 and base : corresponding altitude is 3:4, then the altitude of the triangle

a) 45 b) 36 c) 56 d) None of these

2. The area of a triangle is 216 cm^2 and its sides are in the ratio 3:4:5. Find the perimeter of the triangle.

a) 64 cm b) 72 cm c) 88 cm d) 100 cm

3. One side of a right-angled triangle is twice the other and the hypotenuse is 10 cm. The area of the triangle is

a) 16 cm^2 b) 28 cm^2 c) 20 cm^2 d) 38 cm^2

4. The area of a rhombus is 150 cm2. The length of one of its diagonals is 20 cm. Find the length of the other diagonal.

a) 8 cm b) 15 cm c) 22 cm d) None of these

5. The diagonal of rectangle is thrice its smaller side. Find the ratio of the length to the breadth of the rectangle.

a) $3\sqrt{2}:2$ b) $2\sqrt{2}:3$ c) $2\sqrt{2}:1$ d) None of these

6. Find the circumference of the circle whose area is 16 times the area of the circle with diameter 1.4 m.

a) 15.4 m b) 12.8 m c) 17.6 m d) 22.4 m

7. The ratio between the circumferences of two circles is 4:9. Find the ratio between their areas.

a) 12:17 b) 14:73 c) 16:81 d) 21:23

8. A wheel of radius 40 cm is attached to a smaller wheel of diameter 24 cm. How many revolutions will the smaller wheel make when the large one makes 150 revolutions?

a) 300 b) 500 c) 475 d) 550

9. A sector with central angle 63° cut out from a circle, contains 19.8 cm^2. Find the radius of the circle.

a) 4 cm b) 6 cm c) 7 cm d) 8 cm

10. The sum of radii of two circles is 7 cm and the difference of their circumference is 8 cm. Find the circumference of the circles.

a) 12 cm b) 14 cm c) 16 cm d) 18 cm

11. A gas collecting jar with inner diameter 6 cm and height 25 cm is filled with a gas. Find the quantity of gas contained.

a) 805.6 cm^3 b) 815.6 cm^3 c) 706.5 cm^3 d) 905.6 cm^3

12. What is the volume of a cylinder with radius 8 cm and height 28 cm?

a) 5632 cm^3 b) 6325 cm^3 c) 3265 cm^3 d) 7354 cm^3

13. The height and volume of a cone are 18 cm & 924 cm^3 respectively. Find the radius of cone?

a) 6 cm b) 8 cm c) 7cm d) 10 cm

14. The ratio of the height and the radius of a cone is 2:3 find the radius if volume of the cone is 384 π cm^3

a) 12 cm b) 10 cm c) 8 cm d) 14 cm

15. The curved surface area of a cone is 314 cm^2. Find the slant height, if the radius is 2 cm.

a) 50 cm b) 45 cm c) 48 cm d) 54 cm

16. A hollow hemisphere of radius 50 cm is painted from inner side at the rate 10 paise per cm^2. Find the expenditure ($\pi = 3.14$)?

a) 1520 Rs b) 1570 Rs c) 1580 Rs d) 1600 Rs

17. The surface area of a sphere is 616 cm^2. Find the radius of the sphere

a) 6 cm b) 8 cm c) 7 cm d) 12 cm

18. The radius & height of a cone are 6 cm & 8 cm respectively. Find the curved surface area of the cone.

a) 140 cm^2 b) 120 cm^2 c) 180 cm^2 d) 188.4 cm^2

19. The total surface area of a cylinder is 2464 cm^2. If the radius and height are equal find radius of cylinder.

a) 14 cm b) 12 cm c) 15 cm d) 19 cm

20. A tinmaker convert a cubical metallic box into 10 cylindrical tins. Side of the cube is 50 cm and radius of the cylinder is 7 cm. Find the height of each cylinder so made, if wastage 12% is incurred in the process

a) 20 cm b) 18 cm c) 23 cm d) 40 cm

21. The length, breadth and height of a box area respectively 12 dm, 4 dm and 3 dm, then the length of the greatest rod that can be put in it is:

a) 13 dm b) 16 dm c) 9 dm d) dm

22. If the length, breadth and height of a cuboid are 2m, 2m and 1m respectively, then its surface area (in m^2) is:

a) 8 b) 12 c) 16 d) 24

23. If the length of diagonal of a cube is $4\sqrt{3}$ cm, then the length of its edge is:

a) 2 cm b) 3 cm c) 4 cm d) 6 cm

24. If the height and the radius of a cone are doubled, the volume of the cone becomes:

a) 3 times b) 4 times c) 6 times d) 8 times

25. A solid metal ball of radius 8 cm is melted and cast into smaller balls, each of radius 2 cm. The number of such balls is:

a) 8 b) 16 c) 32 d) 64

26. If a hemi-spherical dome has an inner diameter of 28 m, then its volume (in m^3) is:

a) 6186.60 b) 5749.33 c) 7099.33 d) 7459.33

27. The total surface area (in cm^2) of a solid hemisphere whose diameter is 14 cm, is:

a) 588 b) 392 c) 147 d) 98

28. If a cylindrical rod of iron whose length is 12 times its radius is melted and cast into spherical balls of the same radius, then the number of balls will be:

a) 3 b) 6 c) 9 d) 27

29. If the volume and the surface area of a sphere are numerically the same, then its radius is:

a) 1 unit b) 2 units c) 3 units d) 4 units

30. The volume of a pyramid (in cubic cm) of base area 16 sq.cm and height 9 cm is:

a) 36 b) 48 c) 72 d) 144

31. The dimensions of a cuboid in cm are 16 × 14 × 20. Find its total surface area?

a) 1648 cm^2 b) 1676 cm^2 c) 1748 cm^2 d) 1878 cm^2

32. The total surface area of a cuboid is 166 cm^2. Find its length if breadth and height are 5 cm and 4 cm. respectively.

a) 6 cm b) 7 cm c) 8 cm d) 10 cm

33. Find the quantity of water, in litres, contained in a cuboidal pit with length 7 cm, breadth 5 m and depth 3.6 m

a) 126800 litres b) 120000 litres c) 126000 litres d) 148060 litres

34. The length, breadth and height of a cuboid are 20 cm, 18 cm and 10 cm respectively. Find its volume.

a) 3600 cm^3 b) 3000 cm^3 c) 3050 cm^3 d) 3400 cm^3

35. The side of cube is 60 cm. Find the total surface area of the cube

a) 20000 cm^2 b) 25000 cm^2 c) 21600 cm^2 d) 25200 cm^2

36. Find the side a cube, if its total surface area is 486 cm^2

a) 10 cm b) 4 cm c) 6 cm d) 9 cm

37. What is the volume of the cube with side 5 cm?

a) 125 cm^3 b) 625 cm^3 c) 225 cm^3 d) 115 cm^3

38. The volume of a cube is 512 cm^3. Find the total surface area of the cube

a) 390 cm^2 b) 384 cm^2 c) 314 cm^2 d) 125 cm^2

39. The side of cube is 6 cm. Find its total surface area.

a) 150 cm^2 b) 216 cm^2 c) 420 cm^2 d) 840 cm^2

40. What is the volume of a cylinder with radius 21 cm and height 12 cm?

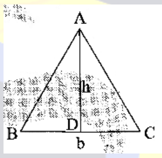
a) 16, 632 cm^3 b) 18834 cm^3 c) 19976 cm^3 d) 22,434 cm^3

Answer Keys

1. c	2. b	3. c	4. b	5. c	6. c	7. c	8. b	9. b	10. d
11. c	12. a	13. c	14. a	15. a	16. b	17. c	18. d	19. a	20. c
21. a	22. c	23. c	24. d	25. d	26. b	27. c	28. c	29. c	30. d
31. a	32. b	33. c	34. a	35. c	36. d	37. a	38. b	39. b	40. a

HINTS AND SOLUTIONS

1. Let BC = base = b, AD = Height = h, Given, b:h = 3:4



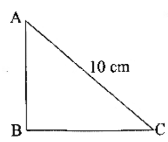
Area of the Δ ABC = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 3x \times 4x$

$\Rightarrow$ 1176 cm^2 = $\frac{1}{2} \times 3x \times 4x = 6x^2$

$\Rightarrow x^2 - \frac{1176}{6} = 196$

$\Rightarrow x = \sqrt{196}$ = 14 cm, Hence, Altitude of the triangle = 4x = 14 × 4 = 56 cm

3. Let ABC is a right-angled triangle.



$\angle B = 90°$

Let other side be x. One side be 2x.

By Pythagoras theorem, $h^2 = p^2 + b^2$

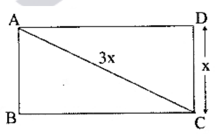
$\Rightarrow (10)^2 = x^2 + (2x)^2 = x^2 + 4x^2$ $\Rightarrow 100 = 5x^2$ $\Rightarrow x^2 = 20$

$\therefore x = \sqrt{20} = 2\sqrt{5}$ cm, BC = x = $2\sqrt{5}$ cm

AB = 2x = $2 \times 2\sqrt{5}$ cm = $4\sqrt{5}$ cm

Area of the triangle = $\frac{1}{2}$ × Base × Height = $\frac{1}{2} \times 2\sqrt{5} \times 4\sqrt{5} = 4 \times 5 = 20$ cm^2

5. Let ABCD is a rectangle in which AB is the length and BC is the breadth.



As given, let smaller side be x.

Diagonal of a rectangle = 3 × Smaller side AC = 3 x x = 3x

In ABC, by Pythagoras theorem, $AC^2 = AB^2 + BC^2 \Rightarrow (3x)^2 = AB^2 + (x)^2$

$\Rightarrow 9x^2 = AB^2 + x^2$ $\Rightarrow 9x^2 - x^2 = AB^2$ $\Rightarrow AB^2 = 8x^2$ $\Rightarrow AB = \sqrt{8x^2} = 2\sqrt{2}\,x$

Hence, Ratio = $\frac{\text{Length}}{Breadth} = \frac{AB}{BC} = \frac{2\sqrt{2}\,x}{x} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}:1$

6. Diameter of the circle = 1.4 m

$r = \frac{d}{2} = \frac{1.4}{2} = 0.7$ m

Diameter of the new circle = x

$r = \frac{d}{2} = \frac{x}{2}$

As per question, Area of the new circle = 16 × Area of the circle

$\pi\left(\frac{x}{2}\right)^2 = 16 \times \pi r^2 = 16 \times \pi \times (0.7)^2$

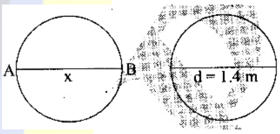
$\frac{x^2}{2^2} = 16 \times 0.49\ m^2 = \frac{16 \times 49\ m^2}{100}$

$x^2 = \frac{16 \times 49\ m^2 \times 4}{100}$

$x = \sqrt{\frac{16 \times 49 \times 4}{100}}\ m^2 = \frac{4 \times 7 \times 2}{10} = 5.6\ m$

Circumference of the circle $2\pi r = 2 \times \frac{22}{7} \times \frac{5.6}{2}\ m = 17.6\ m$

8. b. As given in question, Larger wheel of radius = 40 cm



Circumference of the larger wheel = $2\pi r = 2\pi \times 40$ cm = 80π cm

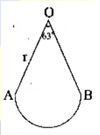
Larger wheel of radius = $\frac{d}{2} = \frac{24}{2} = 12$ cm.

Circumference of the smaller wheel = $2\pi r_1 = 2 \times 12$ cm = 24 cm

Total distance covered by smaller wheel = 80 × 150 revolutions = $24\pi \times x$

$x = \frac{80\pi \times 50}{24\pi} = 500$ revolutions.

9. Area of the sector = $\frac{\pi r^2 \theta}{360°} = \frac{\frac{22}{7} \times r^2 \times 63°}{360°}$



$\Rightarrow r^2 = \frac{19.8\ cm^2 \times 7 \times 360°}{22 \times 63°}$ $\Rightarrow r^2 = 9 \times 4\ cm^2$ ∴ r = 3 × 2 cm = 6 cm

18. $l^2 = r^2 + h^2$ $\Rightarrow 6^2 + 8^2 = 100$, l = 10

curved surface area of the cone,

$S_c = \pi rl = 3.14 \times 6 \times 10 = 188.4\ cm^2$

∴ Answer is (d)

19. Total surface area of a cylinder,

$S_t = 2\pi r\ (h+r)$ ∴ $2464 = \frac{2 \times 22}{7} \times (r\ (r + r)$ [given = r]

∴ $\frac{2464 \times 7}{2 \times 22} = r \times 2r = 2r^2$ ∴ $r^2 = \frac{2464 \times 7}{2 \times 22 \times 2} = 28 \times 7 = 4 \times 7 \times 7$

∴ r = 14 cm. ∴ Answer is (a)

20. side of cubical metallic box = 50 cm.

∴ area of metal sheet = $6l^2 = 6 \times 50 \times 50 = 15000\ cm^2$.

12% of sheet is wasted

∴ sheet wasted = 15000 × $\frac{7}{100}$ = 1800 cm^2

∴ sheet used to prepare tin = 15000 -1800 = 13200 cm^2

from this sheet 10 cylindrical tins are made

∴ sheet used to make 1 cylindrical tin = $\frac{1300}{100}$ = 1320 cm^2 .....(i)

sheet required to make 1 cylindrical tin = total surface area of cylinder

= 2πr (r+ h)= $\frac{2x22}{7}$ × 7 (7+h) .....(ii)

from (1) & (2)

$\frac{2x22}{7}$ × 7 (7+h) = 1320

∴ 7 + h = $\frac{1320}{2x22}$ ∴ 7 + h = 30 ∴ h = 30 - 7 = 23 ∴ Answer is (c)

21. Length of the greatest rod which can be put in the box = length of the diagonal

= $\sqrt{a^2 + b^2 + c^2}$ = $\sqrt{12^2 + 4^2 + 3^2}$ = $\sqrt{144 + 16 + 9}$ = $\sqrt{169}$ = 13dm answer is (a).

22. Surface area of cuboid = 2(lb + bh + hl) = 2(2×2 + 2x1 + 1×2)m^2 = 16m^2

∴ answer is (c).

23. Let the edge of the cube be a cm. Then $\sqrt{3}$a = 4$\sqrt{3}$ ⇒ a = 4. ∴ answer is (c).

24. Let original radius = R and original height = H. New radius = 2R and New height = 2H.

$\frac{\text{New volume}}{\text{Original volume}} = \frac{\frac{1}{2}(2R)^2 \times 2\pi}{\frac{1}{3}\pi R^2 \times H} = \frac{8}{1}$ ∴ So, it becomes 8 times. ∴ answer is (d).

25. Number of Balls = $\frac{\frac{4}{3}\pi \times (8)^3}{\frac{4}{3}\pi \times (2)^3} = \frac{512}{8}$ = 64 ∴ answer is (d).

26. Volume = $\left(\frac{2}{3} x \frac{22}{7} x14x14x14\right)$ = 5749.33m^3 ∴ answer is (b).

27. Total surface area = 3πr^2 = 3 × π ×$(7)^2$ = (147π)cm^2 ∴ answer is (c).

28. Let the radius of the rod be R. Then, its length = 12 R. πR2 ×12R

Number of balls = $\frac{\text{Volume of the rod}}{\text{Volume of 1 ball}} = \frac{\pi R^2 \times 12R}{\frac{4}{3}\pi R^3}$ = 9 ∴ answer is (c).

29. 4πR^2 = $\frac{4}{3}$πR3 ⇒ R = $\left(4 x \frac{3}{4}\right)$ = 3 units ∴ answer is (c).

30. Volume = (Area of the base) x Height = (16 x 9) cm^2 = 144 cm^2 ∴ answer is (d).

33. l = 7m = 700cm; b = 5 m = 500 cm; h = 3.6 m = 360 cm; v = ?

The quantity of water in pit = volume of pit = l × b × h = 700 × 500 × 360 = 126000000 cm^3. Now, 1 litre = 1000 cm^3

∴ the quantity of water in litres = $\frac{126000000}{1000}$ = 126000 litres.

∴ Answer is (c)